



Building AMPL Models into Applications

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INFORMS Annual Meeting

San Francisco — 9-12 November 2014

Session SC56, *Software Demonstrations*

approach (520) business (506) cost (476) data (700) decision (694) develop (569)
dynamic (483) information (704) management (780) method (514)
model (1840) network (748) operations (668)
optimization (1462) present (528) pricing (542)
problem (1291) product (570) programming (514) propose (487)
research (496) results (477) **service** (721) study (704) **systems** (836)



Optimization at ISyE @isyeopt · 17h

Top 25 "interesting" words in the #informs2014 technical program. Created using tagcrowd.com pic.twitter.com/0HVewUCAso

← ↻ 3 ★ 1 ⋮

The Optimization Modeling Cycle

Steps

- ❖ Communicate with problem owner
- ❖ Build model
- ❖ Prepare data
- ❖ Generate optimization problem
- ❖ Submit problem to solver
 - * CPLEX, Gurobi, KNITRO, CONOPT, MINOS, . . .
- ❖ Report & analyze results
- ❖ ***Repeat!***

Goals for optimization software

- ❖ Do this quickly and reliably
- ❖ Get results before client loses interest
- ❖ ***Deploy for application***

Optimization Modeling Languages

Two forms of an optimization problem

- ❖ Modeler's form
 - * Mathematical description, easy for people to work with
- ❖ Algorithm's form
 - * Explicit data structure, easy for solvers to compute with

Idea of a modeling language

- ❖ ***A computer-readable modeler's form***
 - * You write optimization problems in a modeling language
 - * Computers translate to algorithm's form for solution

Advantages of a modeling language

- ❖ Faster modeling cycles
- ❖ More reliable modeling
- ❖ More maintainable applications

Algebraic Modeling Languages

Formulation concept

- ❖ Define data in terms of sets & parameters
 - * Analogous to database keys & records
- ❖ Define decision variables
- ❖ Minimize or maximize a function of decision variables
- ❖ Subject to equations or inequalities that constrain the values of the variables

Advantages

- ❖ Familiar
- ❖ Powerful
- ❖ Implemented

The AMPL Modeling Language

Features

- ❖ Algebraic modeling language
- ❖ Variety of data sources
- ❖ Connections to all solver features

Advantages

- ❖ Powerful, general expressions
- ❖ Natural, easy-to-learn design
- ❖ Efficient processing scales well with problem size

3 ways to use

- ❖ Command language
- ❖ Scripting language
- ❖ Programming interface (API)

Outline

Example: Multicommodity transportation

- ❖ Solution via command language
- ❖ Sensitivity analysis via scripting

Example: Roll cutting

- ❖ Pattern enumeration
 - * via scripting
 - * via MATLAB API
 - * via Java API
- ❖ Pattern generation
 - * via scripting
 - * via MATLAB API

Availability . . .

Command Language

Multicommodity transportation . . .

- ❖ Products available at factories
- ❖ Products needed at stores
- ❖ Plan shipments at lowest cost

. . . with practical restrictions

- ❖ Cost has fixed and variable parts
- ❖ Shipments cannot be too small
- ❖ Factories cannot serve too many stores

Multicommodity Transportation

Given

- O Set of origins (factories)
- D Set of destinations (stores)
- P Set of products

and

- a_{ip} Amount available, for each $i \in O$ and $p \in P$
- b_{jp} Amount required, for each $j \in D$ and $p \in P$
- l_{ij} Limit on total shipments, for each $i \in O$ and $j \in D$
- c_{ijp} Shipping cost per unit, for each $i \in O, j \in D, p \in P$
- d_{ij} Fixed cost for shipping any amount from $i \in O$ to $j \in D$
- s Minimum total size of any shipment
- n Maximum number of destinations served by any origin

Multicommodity Transportation

Mathematical Formulation

Determine

X_{ijp} Amount of each $p \in P$ to be shipped from $i \in O$ to $j \in D$

Y_{ij} 1 if any product is shipped from $i \in O$ to $j \in D$
0 otherwise

to minimize

$$\sum_{i \in O} \sum_{j \in D} \sum_{p \in P} c_{ijp} X_{ijp} + \sum_{i \in O} \sum_{j \in D} d_{ij} Y_{ij}$$

Total variable cost plus total fixed cost

Mathematical Formulation

Subject to

$$\sum_{j \in D} X_{ijp} \leq a_{ip} \quad \text{for all } i \in O, p \in P$$

Total shipments of product p out of origin i
must not exceed availability

$$\sum_{i \in O} X_{ijp} = b_{jp} \quad \text{for all } j \in D, p \in P$$

Total shipments of product p into destination j
must satisfy requirements

Mathematical Formulation

Subject to

$$\sum_{p \in P} X_{ijp} \leq l_{ij} Y_{ij} \quad \text{for all } i \in O, j \in D$$

When there are shipments from origin i to destination j , the total may not exceed the limit, and Y_{ij} must be 1

$$\sum_{p \in P} X_{ijp} \geq s Y_{ij} \quad \text{for all } i \in O, j \in D$$

When there are shipments from origin i to destination j , the total amount of shipments must be at least s

$$\sum_{j \in D} Y_{ij} \leq n \quad \text{for all } i \in O$$

Number of destinations served by origin i must be at most n

AMPL Formulation

Symbolic data

```
set ORIG;    # origins
set DEST;    # destinations
set PROD;    # products

param supply {ORIG,PROD} >= 0; # availabilities at origins
param demand {DEST,PROD} >= 0; # requirements at destinations
param limit  {ORIG,DEST} >= 0; # capacities of links

param vcost  {ORIG,DEST,PROD} >= 0; # variable shipment cost
param fcost  {ORIG,DEST} > 0;      # fixed usage cost

param minload >= 0;                # minimum shipment size
param maxserve integer > 0;       # maximum destinations served
```

AMPL Formulation

Symbolic model: variables and objective

```
var Trans {ORIG,DEST,PROD} >= 0;    # actual units to be shipped
var Use {ORIG, DEST} binary;        # 1 if link used, 0 otherwise

minimize Total_Cost:
    sum {i in ORIG, j in DEST, p in PROD} vcost[i,j,p] * Trans[i,j,p]
+ sum {i in ORIG, j in DEST} fcost[i,j] * Use[i,j];
```

$$\sum_{i \in O} \sum_{j \in D} \sum_{p \in P} c_{ijp} X_{ijp} + \sum_{i \in O} \sum_{j \in D} d_{ij} Y_{ij}$$

Multicommodity Transportation

AMPL Formulation

Symbolic model: constraint

```
subject to Supply {i in ORIG, p in PROD}:  
    sum {j in DEST} Trans[i,j,p] <= supply[i,p];
```

$$\sum_{j \in D} X_{ijp} \leq a_{ip}, \text{ for all } i \in O, p \in P$$

AMPL Formulation

Symbolic model: constraints

```
subject to Supply {i in ORIG, p in PROD}:  
    sum {j in DEST} Trans[i,j,p] <= supply[i,p];  
  
subject to Demand {j in DEST, p in PROD}:  
    sum {i in ORIG} Trans[i,j,p] = demand[j,p];  
  
subject to Multi {i in ORIG, j in DEST}:  
    sum {p in PROD} Trans[i,j,p] <= limit[i,j] * Use[i,j];  
  
subject to Min_Ship {i in ORIG, j in DEST}:  
    sum {p in PROD} Trans[i,j,p] >= minload * Use[i,j];  
  
subject to Max_Serve {i in ORIG}:  
    sum {j in DEST} Use[i,j] <= maxserve;
```


AMPL Formulation

Explicit data independent of symbolic model

```
set ORIG := GARY CLEV PITT ;
set DEST := FRA DET LAN WIN STL FRE LAF ;
set PROD := bands coils plate ;

param supply (tr):  GARY  CLEV  PITT :=
                    bands  400   700   800
                    coils  800  1600  1800
                    plate  200   300   300 ;

param demand (tr):
                    FRA  DET  LAN  WIN  STL  FRE  LAF :=
bands  300  300  100  75  650  225  250
coils  500  750  400  250  950  850  500
plate  100  100   0   50  200  100  250 ;

param limit default 625 ;
param minload := 375 ;
param maxserve := 5 ;
```

AMPL Formulation

Explicit data (continued)

```
param vcost :=
  [*,*,bands]:  FRA  DET  LAN  WIN  STL  FRE  LAF :=
    GARY    30   10   8   10   11   71   6
    CLEV    22   7   10   7   21   82  13
    PITT    19  11  12  10  25   83  15
  [*,*,coils]:  FRA  DET  LAN  WIN  STL  FRE  LAF :=
    GARY    39  14  11  14  16  82   8
    CLEV    27   9  12   9  26  95  17
    PITT    24  14  17  13  28  99  20
  [*,*,plate]:  FRA  DET  LAN  WIN  STL  FRE  LAF :=
    GARY    41  15  12  16  17  86   8
    CLEV    29   9  13   9  28  99  18
    PITT    26  14  17  13  31 104  20 ;
param fcost:    FRA  DET  LAN  WIN  STL  FRE  LAF :=
  GARY  3000 1200 1200 1200 2500 3500 2500
  CLEV  2000 1000 1500 1200 2500 3000 2200
  PITT  2000 1200 1500 1500 2500 3500 2200 ;
```

Multicommodity Transportation

AMPL Solution

Model + data = problem instance to be solved

```
ampl: model multmip3.mod;
ampl: data multmip3.dat;
ampl: option solver gurobi;
ampl: solve;
Gurobi 5.6.0: optimal solution; objective 235625
293 simplex iterations
28 branch-and-cut nodes
ampl: display Use;
Use [*,*]
:      DET FRA FRE LAF LAN STL WIN  :=
CLEV   1   1   1   0   1   1   0
GARY   0   0   0   1   0   1   1
PITT   1   1   1   1   0   1   0
;
```

Multicommodity Transportation

AMPL Solution

Solver choice independent of model and data

```
ampl: model multmip3.mod;
ampl: data multmip3.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.6.0.0: optimal integer solution; objective 235625
136 MIP simplex iterations
0 branch-and-bound nodes
ampl: display Use;
Use [*,*]
:      DET FRA FRE LAF LAN STL WIN  :=
CLEV   1   1   1   0   1   1   0
GARY   0   0   0   1   0   1   1
PITT   1   1   1   1   0   1   0
;
```

Multicommodity Transportation

AMPL Solution

Examine results

```
AMPL: display {i in ORIG, j in DEST}
AMPL?   sum {p in PROD} Trans[i,j,p] / limit[i,j];

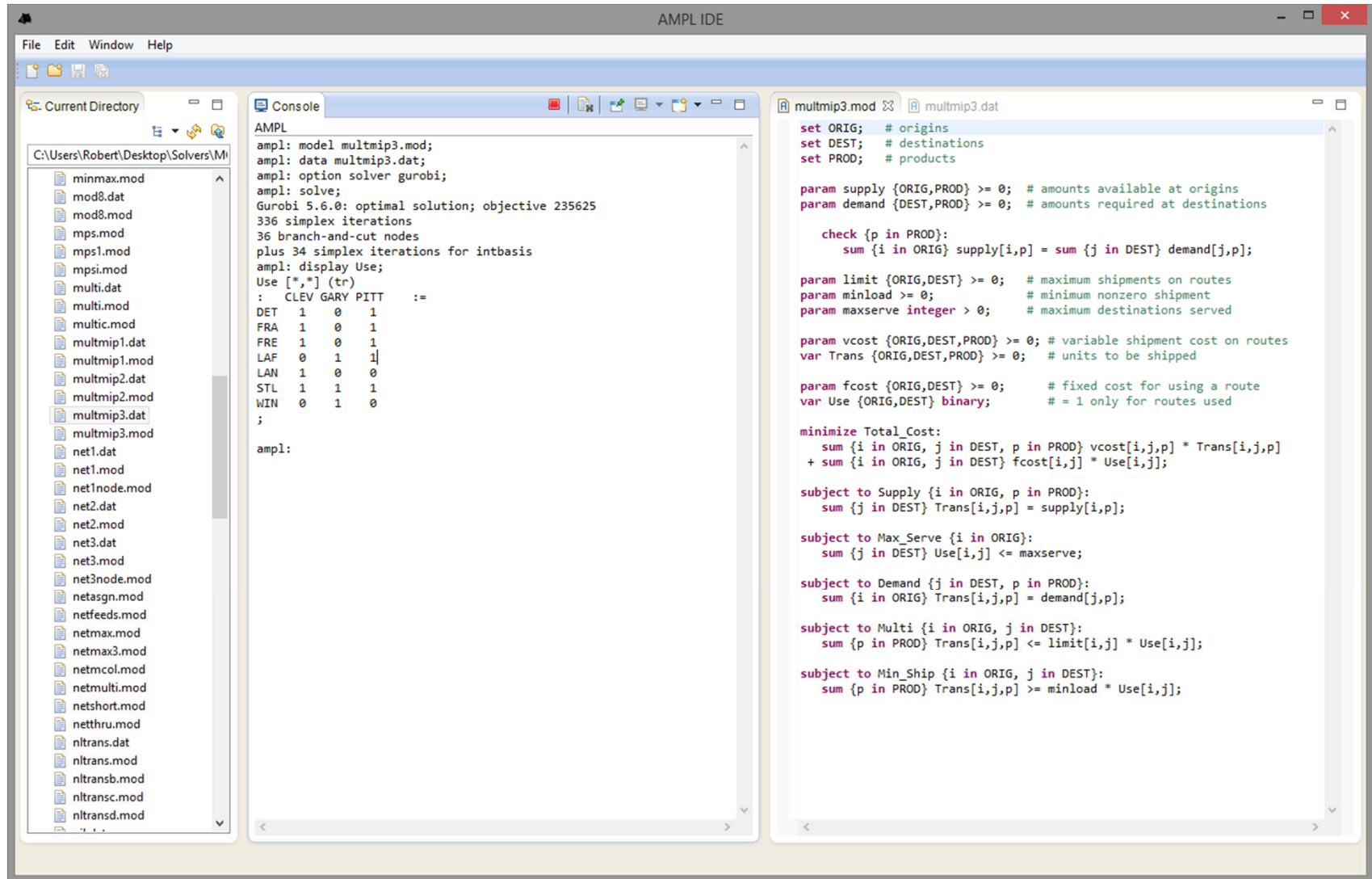
:      DET      FRA      FRE      LAF      LAN      STL      WIN      :=
CLEV   1         0.6     0.88    0        0.8     0.88    0
GARY   0         0       0       0.64    0       1       0.6
PITT   0.84     0.84    1       0.96    0       1       0
;

AMPL: display Max_Serve.body;
CLEV   5
GARY   3
PITT   5
;

AMPL: display TotalCost,
AMPL?   sum {i in ORIG, j in DEST} fcost[i,j] * Use[i,j];
TotalCost = 235625
sum {i in ORIG, j in DEST} fcost[i,j]*Use[i,j] = 27600
```

Multicommodity Transportation

AMPL IDE



Scripting

Extend modeling language syntax . . .

- ❖ Algebraic expressions
- ❖ Set indexing expressions
- ❖ Interactive commands

. . . with programming concepts

- ❖ Loops of various kinds
- ❖ If-then and If-then-else conditionals
- ❖ Assignments

Scripting

Parametric Analyses

Try different limits on destinations served

- ❖ Reduce parameter `maxserve` and re-solve
 - * until there is no feasible solution
- ❖ Display results
 - * parameter value
 - * numbers of destinations actually served

Try different supplies of plate at Gary

- ❖ Increase parameter `supply['GARY', 'plate']` and re-solve
 - * until dual is zero (constraint is slack)
- ❖ Record results
 - * distinct dual values
 - * corresponding objective values

... display results at the end

Scripting

Parametric Analysis *on limits*

Script

```
model multmipG.mod;
data multmipG.dat;

option solver gurobi;

for {m in 7..1 by -1} {
  let maxserve := m;
  solve;
  if solve_result = 'infeasible' then break;
  display maxserve, Max_Serve.body;
}
```

```
subject to Max_Serve {i in ORIG}:
  sum {j in DEST} Use[i,j] <= maxserve;
```

Scripting

Parametric Analysis *on limits*

Run

```
ampl: include multmipServ.run;

Gurobi 5.6.0: optimal solution; objective 233150
maxserve = 7
CLEV 5   GARY 3   PITT 6

Gurobi 5.6.0: optimal solution; objective 233150
maxserve = 6
CLEV 5   GARY 3   PITT 6

Gurobi 5.6.0: optimal solution; objective 235625
maxserve = 5
CLEV 5   GARY 3   PITT 5

Gurobi 5.6.0: infeasible
```

Scripting

Parametric Analysis on supplies

Script

```
set SUPPLY default {};  
param sup_obj {SUPPLY};  
param sup_dual {SUPPLY};  
  
let supply['GARY','plate'] := 200;  
param supply_step = 10;  
param previous_dual default -Infinity;  
  
repeat while previous_dual < 0 {  
  solve;  
  
  if Supply['GARY','plate'].dual > previous_dual then {  
    let SUPPLY := SUPPLY union {supply['GARY','plate']};  
    let sup_obj[supply['GARY','plate']] := Total_Cost;  
    let sup_dual[supply['GARY','plate']] := Supply['GARY','plate'].dual;  
    let previous_dual := Supply['GARY','plate'].dual;  
  }  
  
  let supply['GARY','plate'] := supply['GARY','plate'] + supply_step;  
}
```

Scripting

Parametric Analysis on supplies

Run

```
ampl: include multmipSupply.run;

ampl: display sup_obj, sup_dual;

:      sup_obj    sup_dual    :=
200    223504     -13
380    221171     -11.52
460    220260     -10.52
510    219754     -8.52
560    219413     0
;
```

Scripting

Cutting *via* Pattern Enumeration

Roll cutting

- ❖ Meet orders for small widths by cutting large rolls
 - * using a variety of cutting patterns
- ❖ Decision variables: numbers of each pattern to cut
- ❖ Objective: minimize large rolls used (or material wasted)
- ❖ Constraints: meet demands for each ordered width

Enumerate cutting patterns

- ❖ Read general model
- ❖ Read data: demands, raw width
- ❖ Compute data: all usable patterns
- ❖ Solve problem instance

Scripting

Pattern Enumeration

Model

```
param roll_width > 0;
set WIDTHS ordered by reversed Reals;
param orders {WIDTHS} > 0;

param maxPAT integer >= 0;
param nPAT integer >= 0, <= maxPAT;

param nbr {WIDTHS,1..maxPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:
    sum {j in 1..nPAT} Cut[j];

subj to Fulfill {i in WIDTHS}:
    sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

Scripting

Pattern Enumeration

Data

```
param roll_width := 64.50 ;  
param: WIDTHS: orders :=  
    6.77    10  
    7.56    40  
    17.46   33  
    18.76   10 ;
```

Scripting

Pattern Enumeration

Script (initialize)

```
model cutPAT.mod;
data Sorrentino.dat;

model;
param curr_sum >= 0;
param curr_width > 0;
param pattern {WIDTHS} integer >= 0;

let maxPAT := 100000000;

let nPAT := 0;
let curr_sum := 0;
let curr_width := first(WIDTHS);
let {w in WIDTHS} pattern[w] := 0;
```


Scripting

Pattern Enumeration

Script (loop)

```
repeat {
  if curr_sum + curr_width <= roll_width then {
    let pattern[curr_width] := floor((roll_width-curr_sum)/curr_width);
    let curr_sum := curr_sum + pattern[curr_width] * curr_width;
  }
  if curr_width != last(WIDTHS) then
    let curr_width := next(curr_width,WIDTHS);
  else {
    let nPAT := nPAT + 1;
    let {w in WIDTHS} nbr[w,nPAT] := pattern[w];
    let curr_sum := curr_sum - pattern[last(WIDTHS)] * last(WIDTHS);
    let pattern[last(WIDTHS)] := 0;
    let curr_width := min {w in WIDTHS: pattern[w] > 0} w;
    if curr_width < Infinity then {
      let curr_sum := curr_sum - curr_width;
      let pattern[curr_width] := pattern[curr_width] - 1;
      let curr_width := next(curr_width,WIDTHS);
    }
    else break;
  }
}
```

Scripting

Pattern Enumeration

Script (solve, report)

```
option solver gurobi;
solve;
printf "\n%5i patterns, %3i rolls", nPAT, sum {j in 1..nPAT} Cut[j];
printf "\n\n Cut  ";
printf {j in 1..nPAT: Cut[j] > 0}: "%3i", Cut[j];
printf "\n\n";
for {i in WIDTHS} {
    printf "%7.2f ", i;
    printf {j in 1..nPAT: Cut[j] > 0}: "%3i", nbr[i,j];
    printf "\n";
}
printf "\nWASTE = %5.2f%\n\n",
    100 * (1 - (sum {i in WIDTHS} i * orders[i]) / (roll_width * Number));
```

Scripting

Pattern Enumeration

Results

```
ampl: include cutPatEnum.run
```

```
Gurobi 5.6.0: optimal solution; objective 18
```

```
7 simplex iterations
```

43 patterns, 18 rolls

Cut	2	2	3	11
-----	---	---	---	----

18.76	3	2	0	0
-------	---	---	---	---

17.46	0	1	3	2
-------	---	---	---	---

7.56	1	1	1	3
------	---	---	---	---

6.77	0	0	0	1
------	---	---	---	---

```
WASTE = 2.34%
```

Scripting

Pattern Enumeration

Data 2

```
param roll_width := 349 ;  
param: WIDTHS: orders :=  
    28.75    7  
    33.75    23  
    34.75    23  
    37.75    31  
    38.75    10  
    39.75    39  
    40.75    58  
    41.75    47  
    42.25    19  
    44.75    13  
    45.75    26 ;
```

Scripting

Pattern Enumeration

Results 2

```
ampl: include cutPatEnum.run
```

```
Gurobi 4.6.1: optimal solution; objective 34
```

```
291 simplex iterations
```

```
54508 patterns, 34 rolls
```

Cut	8	1	1	1	3	1	1	1	1	2	7	2	3	1	1
45.75	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0
44.75	1	2	2	1	0	0	0	0	0	0	0	0	0	0	0
42.25	0	2	0	0	4	2	2	1	0	0	0	0	0	0	0
41.75	4	2	0	2	0	0	0	0	2	1	1	0	0	0	0
40.75	0	0	4	4	1	4	3	0	2	3	1	6	3	2	2
39.75	0	0	0	0	0	0	0	2	0	0	5	0	0	2	0
38.75	0	0	1	0	0	0	0	0	4	0	0	0	0	2	3
37.75	0	0	0	0	0	0	1	0	0	4	0	0	6	2	4
34.75	0	0	0	0	4	0	3	1	0	0	0	3	0	1	0
33.75	0	0	0	0	0	3	0	4	0	1	2	0	0	0	0
28.75	0	0	2	2	0	0	0	2	1	0	0	0	0	0	0

```
WASTE = 0.69%
```

Scripting

Pattern Enumeration

Data 3

```
param roll_width := 172 ;  
param: WIDTHS: orders :=  
    25.000    5  
    24.750    73  
    18.000    14  
    17.500    4  
    15.500    23  
    15.375    5  
    13.875    29  
    12.500    87  
    12.250    9  
    12.000    31  
    10.250    6  
    10.125    14  
    10.000    43  
    8.750     15  
    8.500     21  
    7.750     5 ;
```

Scripting

Pattern Enumeration

Results 3 (using a subset of patterns)

```
ampl: include cutPatEnum.run
```

```
Gurobi 4.6.1: optimal solution; objective 33
```

```
722 simplex iterations
```

```
40 branch-and-cut nodes
```

```
273380 patterns, 33 rolls
```

Cut	1	1	1	1	4	4	4	1	1	2	5	2	1	1	1	3
25.00	2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
24.75	1	2	1	0	5	4	3	2	2	2	2	1	1	0	0	0
18.00	0	0	0	0	1	0	0	1	0	0	0	1	1	5	1	0
17.50	0	3	0	0	0	0	0	0	0	0	0	0	0	0	1	0
.....																
10.12	0	2	0	0	0	1	2	0	0	0	0	0	0	0	0	0
10.00	0	0	0	0	0	2	0	1	3	0	6	0	0	2	0	0
8.75	0	0	1	0	0	0	0	0	0	2	0	2	0	0	0	2
8.50	0	0	2	0	0	2	0	0	0	0	0	4	3	0	0	0
7.75	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0

```
WASTE = 0.62%
```

Scripting

Cutting via Pattern Generation

Same roll cutting application

Generate cutting patterns

- ❖ Solve LP relaxation using subset of patterns
- ❖ Add “most promising” pattern to the subset
 - * Minimize reduced cost given dual values
 - * Equivalent to a knapsack problem
- ❖ Iterate as long as there are promising patterns
 - * Stop when minimum reduced cost is zero
- ❖ Solve IP using all patterns found

Scripting

Pattern Generation

Cutting model

```
set WIDTHS ordered by reversed Reals;
param orders {WIDTHS} > 0;

param nPAT integer >= 0, <= maxPAT;
param nbr {WIDTHS,1..nPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:
    sum {j in 1..nPAT} Cut[j];

subj to Fulfill {i in WIDTHS}:
    sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

Scripting

Pattern Generation

Knapsack model

```
param roll_width > 0;
param price {WIDTHS} default 0.0;

var Use {WIDTHS} integer >= 0;

minimize Reduced_Cost:
    1 - sum {i in WIDTHS} price[i] * Use[i];

subj to Width_Limit:
    sum {i in WIDTHS} i * Use[i] <= roll_width;
```

Scripting

Pattern Generation

Script (problems, initial patterns)

```
model cutPatGen.mod;
data Sorrentino.dat;

problem Cutting_Opt: Cut, Number, Fill;
    option relax_integrality 1;
    option presolve 0;

problem Pattern_Gen: Use, Reduced_Cost, Width_Limit;
    option relax_integrality 0;
    option presolve 1;

let nPAT := 0;
for {i in WIDTHS} {
    let nPAT := nPAT + 1;
    let nbr[i,nPAT] := floor (roll_width/i);
    let {i2 in WIDTHS: i2 <> i} nbr[i2,nPAT] := 0;
};
```

Scripting

Pattern Generation

Script (generation loop)

```
repeat {
  solve Cutting_Opt;
  let {i in WIDTHS} price[i] := Fill[i].dual;
  solve Pattern_Gen;
  printf "\n%7.2f%11.2e  ", Number, Reduced_Cost;
  if Reduced_Cost < -0.00001 then {
    let nPAT := nPAT + 1;
    let {i in WIDTHS} nbr[i,nPAT] := Use[i];
  }
  else break;
  for {i in WIDTHS} printf "%3i", Use[i];
};
```

Scripting

Pattern Generation

Script (final integer solution)

```
option Cutting_Opt.relax_integrality 0;
option Cutting_Opt.presolve 10;
solve Cutting_Opt;

if Cutting_Opt.result = "infeasible" then
    printf "\n*** No feasible integer solution ***\n\n";
else {
    printf "Best integer: %3i rolls\n\n", sum {j in 1..nPAT} Cut[j];
    for {j in 1..nPAT: Cut[j] > 0} {
        printf "%3i of:", Cut[j];
        printf {i in WIDTHS: nbr[i,j] > 0}: "%3i x %6.3f", nbr[i,j], i;
        printf "\n";
    }

    printf "\nWASTE = %5.2f%%\n\n",
        100 * (1 - (sum {i in WIDTHS} i * orders[i]) / (roll_width * Number));
}
```

Scripting

Pattern Generation

Results (relaxation)

```
ampl: include cutpatgen.run
```

```
20.44 -1.53e-01 1 3 2 0
18.78 -1.11e-01 0 1 3 0
18.37 -1.25e-01 0 1 0 3
17.96 -4.17e-02 0 6 0 1
17.94 -1.00e-06
```

Optimal relaxation: **17.9412 rolls**

```
10.0000 of: 1 x 6.770 3 x 7.560 2 x 17.460
4.3333 of: 1 x 7.560 3 x 17.460
3.1961 of: 1 x 7.560 3 x 18.760
0.4118 of: 6 x 7.560 1 x 18.760
```

WASTE = 2.02%

Scripting

Pattern Generation

Results (integer)

Rounded up to integer: **20 rolls**

Cut	10	5	4	1
6.77	1	0	0	0
7.56	3	1	1	6
17.46	2	3	0	0
18.76	0	0	3	1

WASTE = 12.10%

Best integer: **19 rolls**

Cut	10	5	3	1
6.77	1	0	0	0
7.56	3	1	1	6
17.46	2	3	0	0
18.76	0	0	3	1

WASTE = 7.48%

Scripting

General Observations

Scripts in practice

- ❖ Large and complicated
 - * Multiple files
 - * Hundreds of statements
 - * Millions of statements executed
- ❖ Run within broader applications

Prospective improvements

- ❖ Faster loops
- ❖ True script functions
 - * Arguments and return values
 - * Local sets & parameters
 - * Callback functions

But . . .

Scripting

Limitations

Performance

- ❖ Interpreted language
- ❖ Complex set & data structures

Expressiveness

- ❖ Based on a declarative language
- ❖ Not object-oriented

So . . .

AMPL API

Application Programming Interface

- ❖ General-purpose languages: C++, Java, .NET, Python
- ❖ Analytics languages: MATLAB, R

Facilitates use of AMPL for

- ❖ Complex algorithmic schemes
- ❖ Embedding in other applications
- ❖ Deployment of models

Development details

- ❖ Partnership with OptiRisk Systems
 - * Christian Valente, principal developer
- ❖ Long-term development & maintenance by AMPL
 - * Victor Zverovich, project coordinator

AMPL API

Cutting Revisited

Hybrid approach

- ❖ Model & modeling commands in AMPL
- ❖ Control & pattern creation from a programming language
 - * Pattern enumeration: finding all patterns
 - * Pattern generation: solving knapsack problems

Two programming languages

- ❖ Java
- ❖ MATLAB

Key to examples

- ❖ AMPL entities
- ❖ Java/MATLAB objects
- ❖ Java/MATLAB methods for working with AMPL

AMPL Model File

Basic pattern-cutting model

```
param nPatterns integer > 0;

set PATTERNS = 1..nPatterns; # patterns
set WIDTHS; # finished widths

param order {WIDTHS} >= 0; # rolls of width j ordered
param overrun; # permitted overrun on any width

param rolls {WIDTHS,PATTERNS} >= 0; # rolls of width i in pattern j

var Cut {PATTERNS} integer >= 0; # raw rolls to cut in each pattern

minimize TotalRawRolls: sum {p in PATTERNS} Cut[p];

subject to FinishedRollLimits {w in WIDTHS}:
    order[w] <= sum {p in PATTERNS} rolls[w,p] * Cut[p] <= order[w] + overrun;
```

Pattern Enumeration in MATLAB

Load & generate data, set up AMPL model

```
function cuttingEnum(dataFile)

% Get data from .mat file: roll_width, overrun, widths, orders
load(dataFile);

% Generate pattern matrix
[widthsDec,ind] = sort(widths,'descend');
patmat = patternEnum(roll_width,widthsDec);
patmat(:,ind) = patmat;

% Initialize and load cutting-stock model from file
AMPL = AMPL();
AMPL.read('cut.mod');
```

Pattern Enumeration in MATLAB

Send data to AMPL

```
% Send scalar values
```

```
AMPL.getParameter('overrun').setValues(overrun);  
AMPL.getParameter('nPatterns').setValues(length(patmat));
```

```
% Send order vector
```

```
WidthOrder = DataFrame(1, 'WIDTHS', 'order');  
WidthOrder.setColumn('WIDTHS', num2cell(widths));  
WidthOrder.setColumn('order', orders);  
AMPL.setData(WidthOrder, 'WIDTHS');
```

```
% Send pattern matrix
```

```
AllPatterns = DataFrame(2, 'WIDTHS', 'PATTERNS', 'rolls');  
AllPatterns.setMatrix(patmat, num2cell(widths), num2cell(1:length(patmat)));  
AMPL.setData(AllPatterns)
```

Pattern Enumeration in MATLAB

Solve and report

```
% Solve
AMPL.setOption('solver' , 'gurobi');
AMPL.solve

% Retrieve solution
CuttingPlan = AMPL.getVariable('Cut').getValues();
cutvec = CuttingPlan.getColumnAsDoubles('val');

% Display solution
cuttingPlot (roll_width, widths, patmat(cutvec>0,:), cutvec(cutvec>0))
```

Pattern Enumeration in MATLAB

Enumeration routine

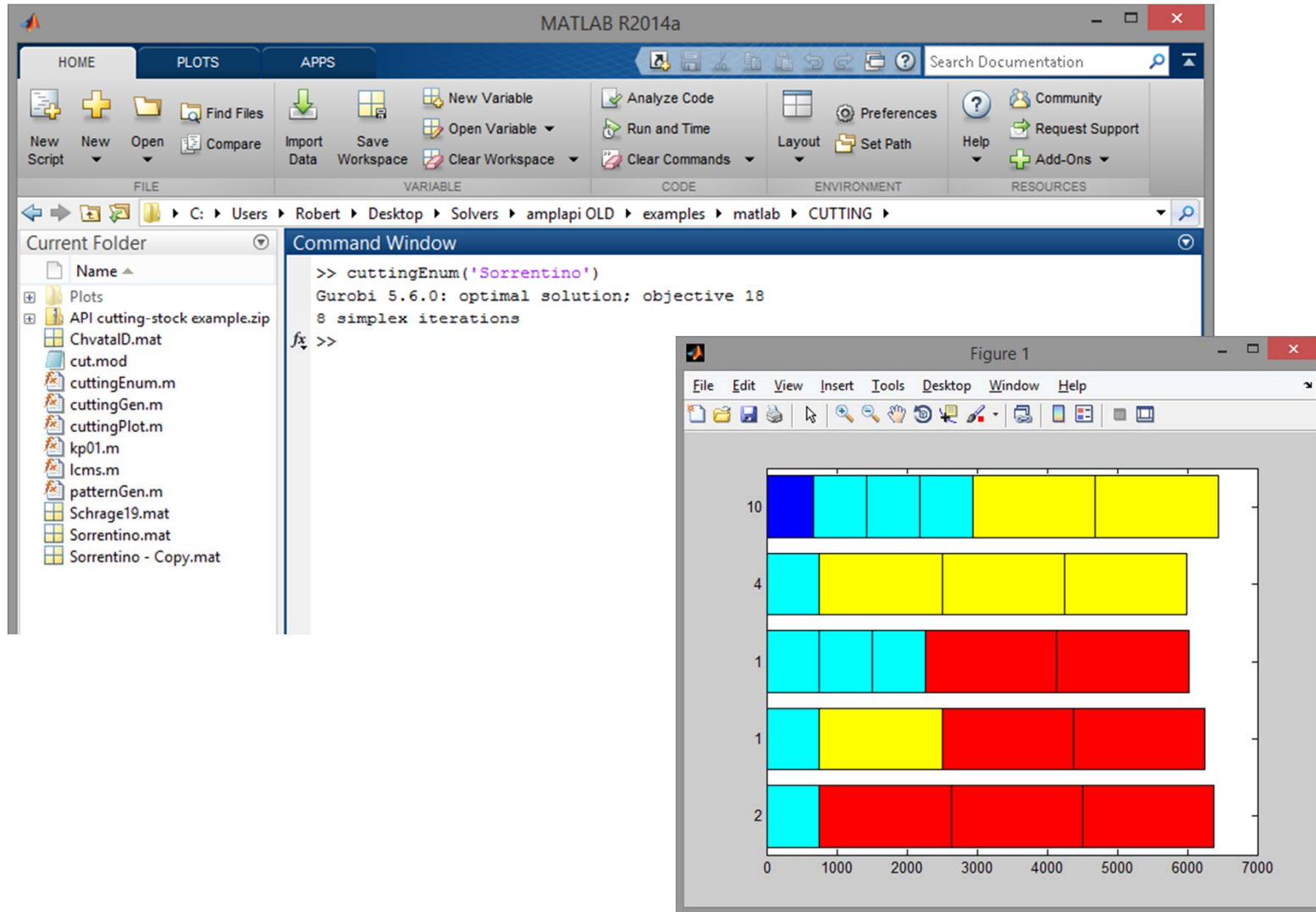
```
function patmat = patternEnum(rollwidth,widths)
if length(widths) == 1
    patmat = floor(rollwidth/widths(1));
else
    patmat = [];
    for n = floor(rollwidth/widths(1)):-1:0
        patnew = patternGen (rollwidth-n*widths(1), widths(2:end));
        patmat = [patmat; n*ones(size(patnew,1),1) patnew];
    end
end
```


Pattern Enumeration in MATLAB

Plotting routine

```
function cuttingPlot (roll_width,widths,patmat,cutvec)
plotmat = zeros(length(cutvec),sum(max(patmat)));
colors = jet(length(widths));
plotpos = 0;
for j = 1:length(widths)
    for i = 1:length(cutvec)
        plotmat(i,plotpos+1:plotpos+patmat(i,j)) = widths(j);
    end
    for i = 1:max(patmat(:,j))
        colormat(plotpos+i,:) = colors(j,:);
    end
    plotpos = plotpos + max(patmat(:,j));
end
colormap(colormat); shading faceted
h = barh(plotmat,'stacked');
set (h, 'edgecolor','black')
set(gca,'YTickLabel',num2cell(cutvec))
```

Pattern Enumeration in MATLAB



Pattern Enumeration in Java

Generate patterns, set up AMPL model

```
public static void main(String[] args) throws IOException {
    import static com.ampl.examples.CuttingStock.Sorrentino;
    int[] sortedWidths = widths.clone();
    sortDescending(sortedWidths);
    ArrayList<Integer> patterns = new ArrayList<>();
    patternEnum (roll_width, sortedWidths, 0, patterns);

    // Initialize and load cutting-stock model from file
    AMPL ampl = new AMPL();
    try {
        ampl.read("cut.mod");
    }
}
```

Pattern Enumeration in Java

Send data to AMPL

```
ampl.getParameter("overrun").setValues(overrun);
int numPatterns = patterns.size() / widths.length;
ampl.getParameter("nPatterns").setValues(numPatterns);

DataFrame widthOrder = new DataFrame(1, "WIDTHS", "order");
widthOrder.setColumn("WIDTHS", widths);
widthOrder.setColumn("order", orders);
ampl.setData(widthOrder, true);

DataFrame allPatterns = new DataFrame(2, "WIDTHS", "PATTERNS", "rolls");
for (int i = 0; i < widths.length; i++) {
    for (int j = 0; j < numPatterns; j++) {
        allPatterns.addRow(
            sortedWidths[i], j + 1, patterns.get(j * widths.length + i));
    }
}
ampl.setData(allPatterns, false);
```

Pattern Enumeration in Java

Solve and report solution

```
    ampl.setOption("solver", "gurobi");
    ampl.solve();
    printSolution (ampl.getVariable("Cut"), ampl.getParameter("rolls"));
} finally {
    ampl.close();
}
}
```

Pattern Generation in MATLAB

Set up AMPL, get data

```
function cuttingGen(dataFile)

% Initialize
AMPL = AMPL();

% Load cutting-stock model from file
AMPL.read('cut.mod');
Cut = AMPL.getVariable('Cut');
Limits = AMPL.getConstraint('FinishedRollLimits');

% Get data from .mat file: roll_width, overrun, widths, orders
load(dataFile);
```

Pattern Generation in MATLAB

Send data to AMPL

```
% Send scalar values
```

```
AMPL.getParameter('overrun').setValues(overrun);  
AMPL.getParameter('nPatterns').setValues(length(widths));
```

```
% Send order vector
```

```
WidthOrder = DataFrame(1, 'WIDTHS', 'order');  
WidthOrder.setColumn('WIDTHS', num2cell(widths));  
WidthOrder.setColumn('order', orders);  
AMPL.setData(WidthOrder, 'WIDTHS');
```

```
% Generate and send initial pattern matrix
```

```
maxpat = floor(roll_width./widths);  
patmat = diag(maxpat);  
InitPatterns = DataFrame(2, 'WIDTHS', 'PATTERNS', 'rolls');  
InitPatterns.setMatrix(patmat, num2cell(widths), num2cell(1:length(widths)));  
AMPL.setData(InitPatterns);
```

Pattern Generation in MATLAB

Set up for generation loop

```
% Set solve options
AMPL.setOption('solver','gurobi');
AMPL.setOption('relax_integrality','1');

% Set up DataFrame for sending AMPL new patterns
AMPL.eval('param newpat {WIDTHS} integer >= 0;');
NewPattern = DataFrame(1, 'WIDTHS', 'newpat');
NewPattern.setColumn('WIDTHS', num2cell(widths));

% Compute multiplier for integer weights
[n,d] = rat(widths);
intmult = lcm(d);
```


Pattern Generation in MATLAB

Loop 1: Retrieve duals & look for new pattern

```
while 1
    ampl.solve
    DualPrices = Limits.getValues;
    dualvec = DualPrices.getColumnAsDoubles('dual');
    wgt = []; val = [];
    for w = 1:length(widths)
        if dualvec(w) > 0
            wgt = [wgt widths(w)*ones(1,maxpat(w))];
            val = [val dualvec(w)*ones(1,maxpat(w))];
        end
    end
    % Solve knapsack problem for potential new pattern
    [kmax,z] = kp01 (round(intmult*wgt), val, intmult*roll_width);
    if kmax < 1.000001
        break;
    end
end
```

Pattern Generation in MATLAB

Loop 2: Send new pattern to AMPL

```
widthlist = wgt(z);
for w = 1:length(widths)
    newpat(w) = length(find(widthlist==widths(w)));
end
patmat = [patmat; newpat];
NewPattern.setColumn('newpat', newpat);
ampl.setData(NewPattern);
ampl.eval('let nPatterns := nPatterns + 1;');
ampl.eval('let {w in WIDTHS} rolls[w,nPatterns] := newpat[w];');
end

% Compute and display integer solution
ampl.setOption('relax_integrality','0');
ampl.solve;

CuttingPlan = Cut.getValues();
cutvec = CuttingPlan.getColumnAsDoubles('val');
cuttingPlot (roll_width, widths, patmat(cutvec>0,:), cutvec(cutvec>0))
```

Pattern Generation in MATLAB

The MATLAB R2014a interface shows the Command Window with the following output:

```
>> cuttingGen('Sorrentino')  
Gurobi 5.6.0: optimal solution; objective 20.44444444  
Gurobi 5.6.0: optimal solution; objective 18.79166667  
Gurobi 5.6.0: optimal solution; obj  
5 simplex iterations  
Gurobi 5.6.0: optimal solution; obj  
5 simplex iterations  
Gurobi 5.6.0: optimal solution; obj  
5 simplex iterations  
Gurobi 5.6.0: optimal solution; obj  
3 simplex iterations  
fx >>
```

The Figure window displays a horizontal bar chart with four bars, each representing a different iteration or solution step. The x-axis ranges from 0 to 70. The bars are composed of segments in cyan, yellow, and red. The segments are arranged as follows:

Iteration	Red	Cyan	Yellow
1	0	0	62
4	0	8	52
10	8	22	30
4	18	0	38

Data Transfer: **Alternatives**

Process

- ❖ Define symbolic sets & parameters in AMPL model
- ❖ Create corresponding objects in program
- ❖ Transfer data using API methods
 - * Program to AMPL
 - * AMPL to program

Methods for transfer between . . .

- ❖ Scalar values
- ❖ Collections of values
 - * AMPL indexed expressions
 - * Java arrays, MATLAB matrices
- ❖ Relational tables
 - * AMPL “table” structures
 - * API DataFrame objects in Java, MATLAB

Deployment: Alternatives

Scripting: Give (temporary) control to AMPL

- ❖ Write needed files
- ❖ Invoke AMPL to run some scripts
- ❖ Read the files that AMPL leaves on exit

API: Interact with AMPL

- ❖ Execute AMPL statements individually
- ❖ Read model, data, script files when convenient
- ❖ Exchange data tables directly with AMPL
 - * populate sets & parameters
 - * invoke any available solver
 - * extract values of variables & result expressions

. . . all embedded within your program's logic

AMPL API

Availability

Best test

- ❖ Java, MATLAB
 - * *Now in progress*
- ❖ C++
 - * *Beginning January 2015*

First release

- ❖ April 2015
- ❖ Available with all AMPL distributions

More languages to follow

- ❖ .NET: C#, Visual Basic
- ❖ Python
- ❖ R

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AMPL
The AMPL system is a sophisticated modeling tool that supports the entire optimization modeling lifecycle: development, testing, deployment, and maintenance.
By using a high-level representation that represents optimization models in the same ways that people think about them, AMPL promotes rapid development and reliable results.
AMPL integrates a modeling language for describing optimization data, variables, objectives, and constraints; a command language for browsing models and analyzing results; and a scripting language for gathering and manipulating data and for implementing iterative

WHAT'S NEW?
March 6-8
6th INFORMS Optimization Society Conference, Houston, Texas.
AMPL sponsorship of this event & table in the exhibit area

AMPL Readings

- ❖ R. Fourer, “Modeling Languages versus Matrix Generators for Linear Programming.” *ACM Transactions on Mathematical Software* **9** (1983) 143–183.
- ❖ R. Fourer, D.M. Gay, B.W. Kernighan, “A Modeling Language for Mathematical Programming.” *Management Science* **36** (1990) 519–554.
- ❖ Robert Fourer, “Database Structures for Mathematical Programming Models.” *Decision Support Systems* **20** (1997) 317–344.
- ❖ R. Fourer, D.M. Gay, B.W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*. Duxbury Press, Belmont, CA (first edition 1993, second edition 2003).
- ❖ Robert Fourer, On the Evolution of Optimization Modeling Systems. M. Groetschel (ed.), *Optimization Stories*. Documenta Mathematica (2012) 377-388.