Developing Optimization Applications Quickly and Effectively with Algebraic Modeling

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Developing Optimization Applications Quickly and Effectively with Algebraic Modeling

Can you negotiate the complexities of the optimization modeling lifecycle, to deliver a working application before the problem owner loses interest? Algebraic languages streamline the key steps of model formulation, testing, and revision, while offering powerful facilities for embedding models into larger systems and deploying them to users. This presentation introduces algebraic modeling for optimization by carrying a single illustrative example through multiple contexts, from interactively evolved formulations to scripted iterative schemes to enterprise applications.

Word cloud, exhibitor descriptions

The Optimization Modeling Cycle

Steps

- Communicate with problem owner
- Build model
- Prepare data
- Generate optimization problem
- Submit problem to solver
  - CPLEX, Gurobi, Xpress, Knitro, CONOPT, MINOS, . . .
- Report & analyze results
- ***Repeat!***

Goals for optimization software

- Do this quickly and reliably
- Get results before client loses interest
- ***Deploy for application***

Optimization Modeling Languages

Two forms of an optimization problem

- ❖ Modeler's form
  - ∗ Mathematical description, easy for people to work with
- ❖ Algorithm's form
  - ∗ Explicit data structure, easy for solvers to compute with

Idea of a modeling language

- ❖ ***A computer-readable modeler's form***
  - ∗ You write optimization problems in a modeling language
  - ∗ Computers translate to algorithm's form for solution

Advantages of a modeling language

- ❖ Faster modeling cycles
- ❖ More reliable modeling
- ❖ More maintainable applications

Algebraic Modeling Languages

Formulation concept

- Define data in terms of sets & parameters
  - Analogous to database keys & records
- Define decision variables
- Minimize or maximize a function of decision variables
- Subject to equations or inequalities
  that constrain the values of the variables

Advantages

- Familiar
- Powerful
- Proven

Categorizations of Algebraic Modeling Languages

By language design

- Extended from a general programming language
- Built specially for optimization

By solver support

- Specialized for one particular solver
- Designed to support many solvers

AMPL

Features

- Algebraic modeling language
- Built specially for optimization
- Designed to support many solvers

Design goals

- Powerful, general expressions
- Natural, easy-to-learn modeling principles
- Efficient processing that scales well with problem size

3 ways to use . . .

3 Ways to Use AMPL

Command language

- ❖ Browse results & debug model interactively
- ❖ Make changes and re-run

Scripting language

- ❖ Bring the programmer to the modeling language

Programming interface (API)

- ❖ Bring the modeling language to the programmer

Example

Roll cutting model

- Solution via command language
- Tradeoff analysis via scripting

Roll cutting by pattern enumeration

- via scripting
- via API

Roll cutting by pattern generation

- via scripting
- via API

In practice . . .

A general tool for applying optimization

- ❖ Based on a broadly applicable paradigm
- ❖ Readily accommodates unanticipated requirements

Ideally positioned for new projects

- ❖ More control
  - ∗ compared to application-specific software
- ❖ Faster, more flexible prototyping
  - ∗ compared to development in a programming language

Scalable for integration and deployment

Roll Cutting Problem

Motivation

- Fill orders for rolls of various widths
  - by cutting raw rolls of one (large) fixed width
  - using a variety of cutting patterns

Optimization model

- Decision variables
  - number of raw rolls to cut according to each pattern
- Objective
  - minimize number of raw rolls used
- Constraints
  - meet demands for each ordered width

Roll cutting

Mathematical Formulation

Given

W	set of ordered widths
n	number of patterns considered

and

a_{ij}	occurrences of width i in pattern j, for each $i \in W$ and $j = 1,\ldots,n$
b_i	orders for width i, for each $i \in W$

Roll cutting

Mathematical Formulation *(cont'd)*

Determine

X_j number of rolls to cut using pattern j,
for each $j = 1, \ldots, n$

to minimize

$\sum_{j=1}^{n} X_j$

total number of rolls cut

subject to

$\sum_{j=1}^{n} a_{ij} X_j \geq b_i$, for all $i \in W$

number of rolls of width i cut
must be at least the number ordered

Roll Cutting

AMPL Formulation

Symbolic model

```
set WIDTHS;
param orders {WIDTHS} > 0;

param nPAT integer >= 0;
param nbr {WIDTHS,1..nPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:

   sum {j in 1..nPAT} Cut[j];

subj to Fulfill {i in WIDTHS}:

   sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

$$\sum_{j=1}^{n} a_{ij} X_j \geq b_i$$

Roll Cutting

AMPL Formulation *(cont'd)*

Explicit data (independent of model)

```
param: WIDTHS: orders :=
         6.77    10
         7.56    40
        17.46    33
        18.76    10 ;

param nPAT := 9 ;

param nbr:  1  2  3  4  5  6  7  8  9 :=
     6.77   0  1  1  0  3  2  0  1  4
     7.56   1  0  2  1  1  4  6  5  2
    17.46   0  1  0  2  1  0  1  1  1
    18.76   3  2  2  1  1  1  0  0  0 ;
```

In practice . . .

Model: decision variables, objective, constraints

- Applicable for many problem types
  - Planning, scheduling, routing, packing, assignment
  - Network flow, portfolio selection, feedstock blending
- Successful in many business areas
  - Production, logistics, sequencing, assignment, design
  - Energy, manufacture, process, finance, commerce

Model + data = Optimization problem for solver

- Model defined & documented independently of data
- Varied data sources supported
  - Text files, spreadsheets, databases, API calls

Command Language

Model + data = problem instance to be solved

```
ampl: model cut.mod;
ampl: data cut.dat;

ampl: option solver cplex;

ampl: solve;

CPLEX 12.6.3.0: optimal integer solution; objective 20
3 MIP simplex iterations

ampl: option omit_zero_rows 1;
ampl: option display_1col 0;

ampl: display Cut;

4 13   7 4   9 3
```

Command Language *(cont'd)*

Solver choice independent of model and data

```
ampl: model cut.mod;
ampl: data cut.dat;

ampl: option solver gurobi;

ampl: solve;

Gurobi 6.5.0: optimal solution; objective 20
3 simplex iterations

ampl: option omit_zero_rows 1;
ampl: option display_1col 0;

ampl: display Cut;

4 13   7 4   9 3
```

Command Language *(cont'd)*

Results available for browsing

```
ampl: display {j in 1..nPAT, i in WIDTHS: Cut[j] > 0} nbr[i,j];

:        4   7   9   :=                                  # patterns used
6.77     0   0   4
7.56     1   6   2
17.46    2   1   1
18.76    1   0   0

ampl: display {j in 1..nPAT} sum {i in WIDTHS} i * nbr[i,j];

1 63.84   3 59.41   5 64.09   7 62.82   9 59.66          # pattern
2 61.75   4 61.24   6 62.54   8 62.0                     # total widths

ampl: display Fulfill.slack;

 6.77  2                                                 # overruns
 7.56  3
17.46  0
18.76  3
```

Roll Cutting

Revision 1

Symbolic model

```
param roll_width > 0;

set WIDTHS;
param orders {WIDTHS} > 0;

param nPAT integer >= 0;
param nbr {WIDTHS,1..nPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:
   sum {j in 1..nPAT} Cut[j];

minimize Waste:
   sum {j in 1..nPAT}
      Cut[j] * (roll_width - sum {i in WIDTHS} i * nbr[i,j]);

subj to Fulfill {i in WIDTHS}:

   sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

Revision 1 *(cont'd)*

Explicit data

```
param roll_width := 64.5;

param: WIDTHS: orders :=
         6.77     10
         7.56     40
        17.46     33
        18.76     10 ;

param nPAT := 9 ;

param nbr:  1  2  3  4  5  6  7  8  9 :=
     6.77   0  1  1  0  3  2  0  1  4
     7.56   1  0  2  1  1  4  6  5  2
    17.46   0  1  0  2  1  0  1  1  1
    18.76   3  2  2  1  1  1  0  0  0 ;
```

Revision 1 *(cont'd)*

Solutions

```
ampl: model cutRev1.mod;
ampl: data cutRev1.dat;

ampl: objective Number; solve;

Gurobi 6.5.0: optimal solution; objective 20
3 simplex iterations

ampl: display Number, Waste;
Number = 20
Waste = 63.62

ampl: objective Waste; solve;

Gurobi 6.5.0: optimal solution; objective 15.62
2 simplex iterations

ampl: display Number, Waste;
Number = 35
Waste = 15.62
```

Roll Cutting

Revision 2

Symbolic model

```
param roll_width > 0;
param over_lim integer >= 0;

set WIDTHS;
param orders {WIDTHS} > 0;

param nPAT integer >= 0;
param nbr {WIDTHS,1..nPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

...

subj to Fulfill {i in WIDTHS}:
   orders[i] <= sum {j in 1..nPAT} nbr[i,j] * Cut[j]
             <= orders[i] + over_lim;
```

Revision 2 *(cont'd)*

Explicit data

```
param roll_width := 64.5;
param over_lim := 6 ;

param: WIDTHS: orders :=
         6.77    10
         7.56    40
        17.46    33
        18.76    10 ;

param nPAT := 9 ;

param nbr:  1  2  3  4  5  6  7  8  9 :=
     6.77   0  1  1  0  3  2  0  1  4
     7.56   1  0  2  1  1  4  6  5  2
    17.46   0  1  0  2  1  0  1  1  1
    18.76   3  2  2  1  1  1  0  0  0 ;
```

Revision 2 *(cont'd)*

Solutions

```
ampl: model cutRev2.mod;
ampl: data cutRev2.dat;

ampl: objective Number; solve;

Gurobi 6.0.4: optimal solution; objective 20
8 simplex iterations

ampl: display Number, Waste;
Number = 20
Waste = 63.62

ampl: objective Waste; solve;

Gurobi 6.0.4: optimal solution; objective 49.16
2 simplex iterations

ampl: display Number, Waste;
Number = 21
Waste = 49.16
```

Further revisions

Overruns

- Limit to percentage of amount ordered
- Limit total extra rolls

Pattern restrictions

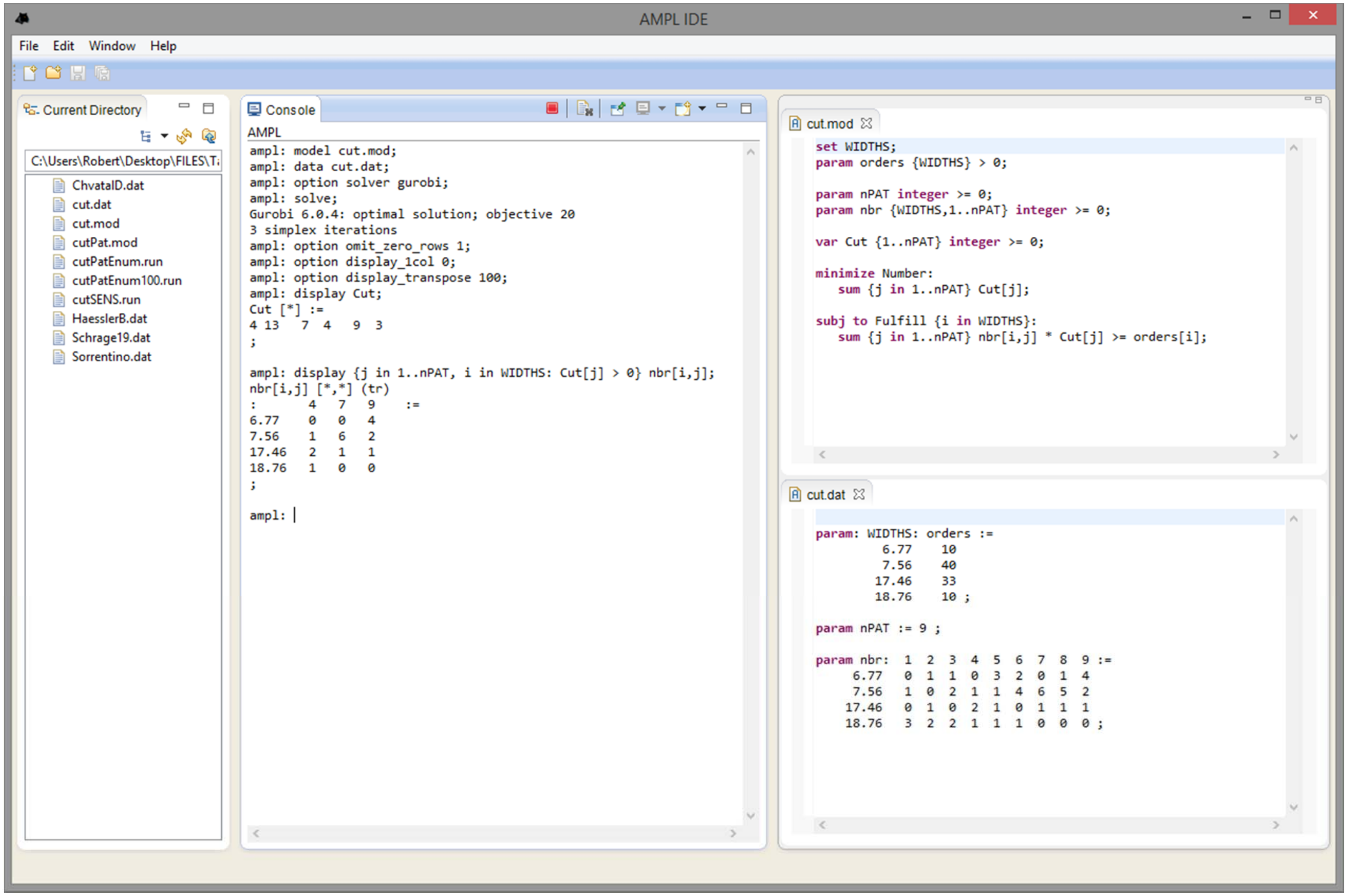
- Cut at least a specified number of each pattern used
- Limit the number of patterns used

Costs

- Account for setups
- Account for complications of cutting

Anything else you can imagine . . .

IDE for Command Language



```
ampl: model cut.mod;
ampl: data cut.dat;
ampl: option solver gurobi;
ampl: solve;
Gurobi 6.0.4: optimal solution; objective 20
3 simplex iterations
ampl: option omit_zero_rows 1;
ampl: option display_1col 0;
ampl: option display_transpose 100;
ampl: display Cut;
Cut [*] :=
4 13   7  4    9  3
;

ampl: display {j in 1..nPAT, i in WIDTHS: Cut[j] > 0} nbr[i,j];
nbr[i,j] [*,*] (tr)
:       4    7    9    :=
6.77    0    0    4
7.56    1    6    2
17.46   2    1    1
18.76   1    0    0
;

ampl: |
```

```
set WIDTHS;
param orders {WIDTHS} > 0;

param nPAT integer >= 0;
param nbr {WIDTHS,1..nPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:
   sum {j in 1..nPAT} Cut[j];

subj to Fulfill {i in WIDTHS}:
   sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

```
param: WIDTHS: orders :=
         6.77     10
         7.56     40
        17.46     33
        18.76     10 ;

param nPAT := 9 ;

param nbr:  1  2  3  4  5  6  7  8  9 :=
     6.77   0  1  1  0  3  2  0  1  4
     7.56   1  0  2  1  1  4  6  5  2
    17.46   0  1  0  2  1  0  1  1  1
    18.76   3  2  2  1  1  1  0  0  0 ;
```

In practice . . .

Work interactively

- Make changes
- Solve
- Browse results
- *Review and repeat*

Choose the best solver for your problem

- Linear solvers
  - CPLEX, Gurobi, Xpress
- Nonlinear solvers
  - CONOPT, Knitro, MINOS, SNOPT
- Alternative solvers
  - BARON, LGO, LocalSolver

Scripting

Bring the programmer to the modeling language

Extend modeling language syntax . . .

- ❖ Algebraic expressions
- ❖ Set indexing expressions
- ❖ Interactive commands

. . . with programming concepts

- ❖ Loops of various kinds
- ❖ If-then and If-then-else conditionals
- ❖ Assignments

Examples

- ❖ Tradeoffs between objectives
- ❖ Cutting *via* pattern enumeration
- ❖ Cutting *via* pattern generation

Scripting

Tradeoffs Between Objectives

Minimize rolls cut

- ❖ Set large overrun limit

Minimize waste

- ❖ Reduce overrun limit 1 roll at a time
- ❖ If there is a change in number of rolls cut
  - ∗ record total waste (increasing)
  - ∗ record total rolls cut (decreasing)
- ❖ Stop when no further progress possible
  - ∗ problem becomes infeasible
  - ∗ total rolls cut falls to the minimum
- ❖ Report table of results

Scripting

Parametric Analysis *(cont'd)*

Script (setup and initial solve)

```
model cutRev2.mod;
data cutRev2.dat;

set OVER default {} ordered by reversed Integers;

param minNumber;
param minNumWaste;
param minWaste {OVER};
param minWasteNum {OVER};

param prev_number default Infinity;

option solver Gurobi;
option solver_msg 0;


objective Number;
solve >Nul;

let minNumber := Number;
let minNumWaste := Waste;

objective Waste;
```

Scripting

Parametric Analysis *(cont'd)*

Script (looping and reporting)

```
for {k in over_lim .. 0 by -1} {
   let over_lim := k;

   solve >Nul;

   if solve_result = 'infeasible' then break;

   if Number < prev_number then {
      let OVER := OVER union {k};
      let minWaste[k] := Waste;
      let minWasteNum[k] := Number;
      let prev_number := Number;
   }

   if Number = minNumber then break;

}

printf 'Min%3d rolls with waste%6.2f\n\n', minNumber, minNumWaste;
printf ' Over  Waste  Number\n';
printf {k in OVER}: '%4d%8.2f%6d\n', k, minWaste[k], minWasteNum[k];
```

Scripting

Parametric Analysis *(cont'd)*

Script run

```
ampl: include cutWASTE.run

Min 20 rolls with waste 63.62

 Over  Waste  Number
  10   46.72    22
   7   47.89    21
   5   54.76    20

ampl:
```

Cutting *via* Pattern Enumeration

Build the pattern list, then solve

- ❖ Read general model
- ❖ Read data: demands, raw width
- ❖ Compute data: all usable patterns
- ❖ Solve problem instance

Scripting

Pattern Enumeration

Model

```
param roll_width > 0;

set WIDTHS ordered by reversed Reals;
param orders {WIDTHS} > 0;

param maxPAT integer >= 0;
param nPAT integer >= 0, <= maxPAT;

param nbr {WIDTHS,1..maxPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:

   sum {j in 1..nPAT} Cut[j];

subj to Fulfill {i in WIDTHS}:

   sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

Scripting

Pattern Enumeration

Data

```
param roll_width := 64.50 ;

param: WIDTHS: orders :=

         6.77    10
         7.56    40
        17.46    33
        18.76    10 ;
```

Scripting

Pattern Enumeration

Script (initialize)

```
model cutPAT.mod;
data Sorrentino.dat;

param curr_sum >= 0;
param curr_width > 0;
param pattern {WIDTHS} integer >= 0;

let maxPAT := 1000000;

let nPAT := 0;
let curr_sum := 0;
let curr_width := first(WIDTHS);
let {w in WIDTHS} pattern[w] := 0;
```

Scripting

Pattern Enumeration

Script (loop)

```
repeat {
   if curr_sum + curr_width <= roll_width then {
      let pattern[curr_width] := floor((roll_width-curr_sum)/curr_width);
      let curr_sum := curr_sum + pattern[curr_width] * curr_width;
      }
   if curr_width != last(WIDTHS) then
      let curr_width := next(curr_width,WIDTHS);
   else {
      let nPAT := nPAT + 1;
      let {w in WIDTHS} nbr[w,nPAT] := pattern[w];
      let curr_sum := curr_sum - pattern[last(WIDTHS)] * last(WIDTHS);
      let pattern[last(WIDTHS)] := 0;
      let curr_width := min {w in WIDTHS: pattern[w] > 0} w;
      if curr_width < Infinity then {
         let curr_sum := curr_sum - curr_width;
         let pattern[curr_width] := pattern[curr_width] - 1;
         let curr_width := next(curr_width,WIDTHS);
         }
      else break;
      }
   }
```

Pattern Enumeration

Script (solve, report)

```
option solver gurobi;

solve;

printf "\n%5i patterns, %3i rolls", nPAT, sum {j in 1..nPAT} Cut[j];
printf "\n\n  Cut   ";
printf {j in 1..nPAT: Cut[j] > 0}: "%3i", Cut[j];
printf "\n\n";

for {i in WIDTHS} {
   printf "%7.2f ", i;
   printf {j in 1..nPAT: Cut[j] > 0}: "%3i", nbr[i,j];
   printf "\n";
   }

printf "\nWASTE = %5.2f%%\n\n",
   100 * (1 - (sum {i in WIDTHS} i * orders[i]) / (roll_width * Number));
```

Pattern Enumeration

Results

```
ampl: include cutPatEnum.run

Gurobi 5.6.0: optimal solution; objective 18
7 simplex iterations

43 patterns, 18 rolls

  Cut      2  2  3 11

  18.76    3  2  0  0
  17.46    0  1  3  2
   7.56    1  1  1  3
   6.77    0  0  0  1

WASTE =  2.34%
```

Scripting

Pattern Enumeration

Data 2

```
param roll_width := 349 ;

param: WIDTHS: orders :=
       28.75     7
       33.75    23
       34.75    23
       37.75    31
       38.75    10
       39.75    39
       40.75    58
       41.75    47
       42.25    19
       44.75    13
       45.75    26 ;
```

Scripting

Pattern Enumeration

Results 2

```
ampl: include cutPatEnum.run

Gurobi 4.6.1: optimal solution; objective 34
291 simplex iterations

54508 patterns,  34 rolls

  Cut      8  1  1  1  3  1  1  1  1  2  7  2  3  1  1

  45.75    3  2  0  0  0  0  0  0  0  0  0  0  0  0  0
  44.75    1  2  2  1  0  0  0  0  0  0  0  0  0  0  0
  42.25    0  2  0  0  4  2  2  1  0  0  0  0  0  0  0
  41.75    4  2  0  2  0  0  0  0  2  1  1  0  0  0  0
  40.75    0  0  4  4  1  4  3  0  2  3  1  6  3  2  2
  39.75    0  0  0  0  0  0  0  2  0  0  5  0  0  2  0
  38.75    0  0  1  0  0  0  0  0  4  0  0  0  0  2  3
  37.75    0  0  0  0  0  0  1  0  0  4  0  0  6  2  4
  34.75    0  0  0  0  4  0  3  1  0  0  0  3  0  1  0
  33.75    0  0  0  0  0  3  0  4  0  1  2  0  0  0  0
  28.75    0  0  2  2  0  0  0  2  1  0  0  0  0  0  0

WASTE =  0.69%
```

Scripting

Pattern Enumeration

Data 3

```
param roll_width := 172 ;

param: WIDTHS: orders :=
       25.000     5
       24.750    73
       18.000    14
       17.500     4
       15.500    23
       15.375     5
       13.875    29
       12.500    87
       12.250     9
       12.000    31
       10.250     6
       10.125    14
       10.000    43
        8.750    15
        8.500    21
        7.750     5 ;
```

Scripting

Pattern Enumeration

Results 3 (using a subset of patterns)

```
ampl: include cutPatEnum.run

Gurobi 4.6.1: optimal solution; objective 33
722 simplex iterations
40 branch-and-cut nodes

273380 patterns,  33 rolls

  Cut     1  1  1  1  4  4  4  1  1  2  5  2  1  1  1  3

  25.00   2  1  1  1  0  0  0  0  0  0  0  0  0  0  0  0
  24.75   1  2  1  0  5  4  3  2  2  2  2  1  1  0  0  0
  18.00   0  0  0  0  1  0  0  1  0  0  0  1  1  5  1  0
  17.50   0  3  0  0  0  0  0  0  0  0  0  0  0  0  1  0
  .......
  10.12   0  2  0  0  0  1  2  0  0  0  0  0  0  0  0  0
  10.00   0  0  0  0  0  2  0  1  3  0  6  0  0  2  0  0
   8.75   0  0  1  0  0  0  0  0  0  2  0  2  0  0  0  2
   8.50   0  0  2  0  0  2  0  0  0  0  0  4  3  0  0  0
   7.75   0  0  0  0  1  0  0  1  0  0  0  0  0  0  0  0

WASTE =  0.62%
```

Cutting *via* Pattern Generation

Generate the pattern list by a series of solves

- ❖ Solve LP relaxation using subset of patterns
- ❖ Add "most promising" pattern to the subset
  - ∗ Minimize reduced cost given dual values
  - ∗ Equivalent to a knapsack problem
- ❖ Iterate as long as there are promising patterns
  - ∗ Stop when minimum reduced cost is zero
- ❖ Solve IP using all patterns found

Scripting

Pattern Generation

Cutting model

```
set WIDTHS ordered by reversed Reals;
param orders {WIDTHS} > 0;

param nPAT integer >= 0, <= maxPAT;
param nbr {WIDTHS,1..nPAT} integer >= 0;

var Cut {1..nPAT} integer >= 0;

minimize Number:

   sum {j in 1..nPAT} Cut[j];

subj to Fulfill {i in WIDTHS}:

   sum {j in 1..nPAT} nbr[i,j] * Cut[j] >= orders[i];
```

Scripting

Pattern Generation

Knapsack model

```
param roll_width > 0;
param price {WIDTHS} default 0.0;

var Use {WIDTHS} integer >= 0;

minimize Reduced_Cost:
   1 - sum {i in WIDTHS} price[i] * Use[i];

subj to Width_Limit:
   sum {i in WIDTHS} i * Use[i] <= roll_width;
```

Scripting

Pattern Generation

Script (problems, initial patterns)

```
model cutPatGen.mod;
data Sorrentino.dat;

problem Cutting_Opt: Cut, Number, Fill;
   option relax_integrality 1;
   option presolve 0;

problem Pattern_Gen: Use, Reduced_Cost, Width_Limit;
   option relax_integrality 0;
   option presolve 1;

let nPAT := 0;

for {i in WIDTHS} {
   let nPAT := nPAT + 1;
   let nbr[i,nPAT] := floor (roll_width/i);
   let {i2 in WIDTHS: i2 <> i} nbr[i2,nPAT] := 0;
   };
```

Scripting

Pattern Generation

Script (generation loop)

```
repeat {
   solve Cutting_Opt;

   let {i in WIDTHS} price[i] := Fill[i].dual;

   solve Pattern_Gen;

   printf "\n%7.2f%11.2e  ", Number, Reduced_Cost;

   if Reduced_Cost < -0.00001 then {
      let nPAT := nPAT + 1;
      let {i in WIDTHS} nbr[i,nPAT] := Use[i];
   }
   else break;

   for {i in WIDTHS} printf "%3i", Use[i];
};
```

Scripting

Pattern Generation

Script (final integer solution)

```
option Cutting_Opt.relax_integrality 0;
option Cutting_Opt.presolve 10;
solve Cutting_Opt;

if Cutting_Opt.result = "infeasible" then
   printf "\n*** No feasible integer solution ***\n\n";
else {
   printf "Best integer: %3i rolls\n\n", sum {j in 1..nPAT} Cut[j];
   for {j in 1..nPAT: Cut[j] > 0} {
      printf "%3i of:", Cut[j];
      printf {i in WIDTHS: nbr[i,j] > 0}: "%3i x %6.3f", nbr[i,j], i;
      printf "\n";
      }
   printf "\nWASTE = %5.2f%%\n\n",
      100 * (1 - (sum {i in WIDTHS} i * orders[i]) / (roll_width * Number));
   }
```

Scripting

Pattern Generation

Results (relaxation)

```
ampl: include cutpatgen.run

  20.44  -1.53e-01    1  3  2  0
  18.78  -1.11e-01    0  1  3  0
  18.37  -1.25e-01    0  1  0  3
  17.96  -4.17e-02    0  6  0  1
  17.94  -1.00e-06

Optimal relaxation: 17.9412 rolls

 10.0000 of:  1 x 6.770  3 x  7.560  2 x 17.460
  4.3333 of:  1 x 7.560  3 x 17.460
  3.1961 of:  1 x 7.560  3 x 18.760
  0.4118 of:  6 x 7.560  1 x 18.760

WASTE =  2.02%
```

Scripting

Pattern Generation

Results (integer)

```
Rounded up to integer: 20 rolls

  Cut     10  5  4  1

   6.77    1  0  0  0
   7.56    3  1  1  6
  17.46    2  3  0  0
  18.76    0  0  3  1

WASTE = 12.10%

Best integer: 19 rolls

  Cut     10  5  3  1

   6.77    1  0  0  0
   7.56    3  1  1  6
  17.46    2  3  0  0
  18.76    0  0  3  1

WASTE =  7.48%
```

In practice . . .

Large and complex scripts

- ❖ Multiple files
- ❖ Hundreds of statements
- ❖ Millions of statements executed

Coordination with enterprise systems

- ❖ Your system
  - ∗ writes data files
  - ∗ invokes `ampl optapp.run`
- ❖ AMPL's script
  - ∗ reads the data files
  - ∗ processes data, generates problems, invokes solvers
  - ∗ writes result files
- ❖ Your system
  - ∗ reads the result files

Scripting

Limitations

Scripts can be slow

- ❖ Interpreted, not compiled
- ❖ Very general set & data structures

Script programming constructs are limited

- ❖ Based on a declarative language
- ❖ Not object-oriented

Scripts are stand-alone

- ❖ Close AMPL environment before returning to system

So . . .

APIs (application programming interfaces)

Bring the modeling language to the programmer

- Data and result management in a general-purpose programming language
- Modeling and solving through calls to AMPL

Development details

- Partnership with OptiRisk Systems
  - Christian Valente, principal developer
- Long-term development & maintenance by AMPL
  - Victor Zverovich, project coordinator

Cutting Revisited

Hybrid approach

- Control & pattern creation from a programming language
  - Pattern enumeration: finding all patterns
  - Pattern generation: solving knapsack problems
- Model & modeling commands in AMPL

Choice of programming languages

- Java, MATLAB, C++

Key to program examples

- AMPL entities
- objects
- methods for working with AMPL
- functions

AMPL API

AMPL Model File

Basic pattern-cutting model

```
param nPatterns integer > 0;

set PATTERNS = 1..nPatterns;  # patterns
set WIDTHS;                   # finished widths

param order {WIDTHS} >= 0;    # rolls of width j ordered
param overrun;                # permitted overrun on any width

param rolls {WIDTHS,PATTERNS} >= 0;  # rolls of width i in pattern j

var Cut {PATTERNS} integer >= 0;     # raw rolls to cut in each pattern

minimize TotalRawRolls: sum {p in PATTERNS} Cut[p];

subject to FinishedRollLimits {w in WIDTHS}:
   order[w] <= sum {p in PATTERNS} rolls[w,p] * Cut[p] <= order[w] + overrun;
```

AMPL API

Pattern Enumeration in MATLAB

Load & generate data, set up AMPL model

```
function cuttingEnum(dataFile)

% Get data from .mat file: roll_width, overrun, widths, orders
load(dataFile);

% Generate pattern matrix
[widthsDec,ind] = sort(widths,'descend');
patmat = patternEnum(roll_width,widthsDec);
patmat(:,ind) = patmat;

% Initialize and load cutting-stock model from file
ampl = AMPL();
ampl.read('cut.mod');
```

Pattern Enumeration in MATLAB

Send data to AMPL

```
% Send scalar values

ampl.getParameter('overrun').setValues(overrun);
ampl.getParameter('nPatterns').setValues(length(patmat));

% Send order vector

WidthOrder = DataFrame(1, 'WIDTHS', 'order');
WidthOrder.setColumn('WIDTHS', num2cell(widths));
WidthOrder.setColumn('order', orders);

ampl.setData(WidthOrder, 'WIDTHS');

% Send pattern matrix

AllPatterns = DataFrame(2, 'WIDTHS', 'PATTERNS', 'rolls');
AllPatterns.setMatrix(patmat',num2cell(widths),num2cell(1:length(patmat)));

ampl.setData(AllPatterns)
```

Pattern Enumeration in MATLAB

Solve and report

```
% Solve

ampl.setOption('solver' ,'gurobi');
ampl.solve

% Retrieve solution

CuttingPlan = ampl.getVariable('Cut').getValues();
cutvec = CuttingPlan.getColumnAsDoubles('val');

% Display solution

cuttingPlot (roll_width, widths, patmat(cutvec>0,:), cutvec(cutvec>0))
```

AMPL API

Pattern Enumeration in MATLAB

Enumeration routine

```
function patmat = patternEnum(rollwidth,widths)

if length(widths) == 1

    patmat = floor(rollwidth/widths(1));

else

   patmat = [];

   for n = floor(rollwidth/widths(1)):-1:0

       patnew = patternEnum (rollwidth-n*widths(1), widths(2:end));
       patmat = [patmat; n*ones(size(patnew,1),1) patnew];

   end

end
```

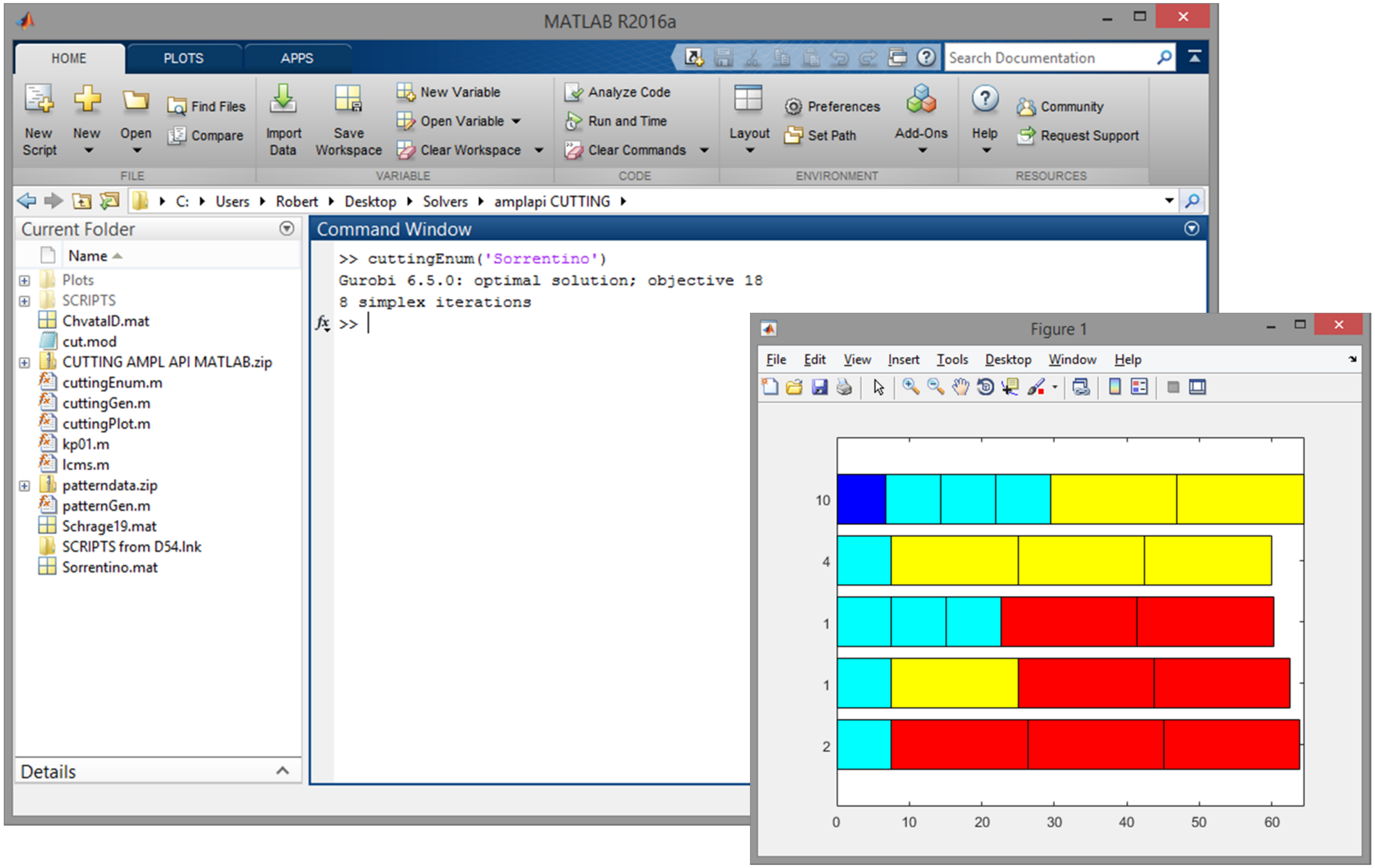
Pattern Enumeration in MATLAB

Plotting routine

```
function cuttingPlot (roll_width,widths,patmat,cutvec)

plotmat = zeros(length(cutvec),sum(max(patmat)));
colors = jet(length(widths));

plotpos = 0;

for j = 1:length(widths)
   for i = 1:length(cutvec)
      plotmat(i,plotpos+1:plotpos+patmat(i,j)) = widths(j);
   end
   for i = 1:max(patmat(:,j))
      colormat(plotpos+i,:) = colors(j,:);
   end
   plotpos = plotpos + max(patmat(:,j));
end

colormap(colormat); shading faceted
h = barh(plotmat,'stacked');

set (h, 'edgecolor','black'); set(gca,'YTickLabel',num2cell(cutvec))
xlim([0,roll_width]); ylim([0,numel(get(gca,'YTick'))+1])
```

AMPL API

Pattern Enumeration in MATLAB



```
>> cuttingEnum('Sorrentino')
Gurobi 6.5.0: optimal solution; objective 18
8 simplex iterations
>>
```

Pattern Enumeration in Java

Generate patterns, set up AMPL model

```
public static void main(String[] args) throws IOException {

  import static com.ampl.examples.CuttingStock.Sorrentino;

  int[] sortedWidths = widths.clone();
  sortDescending(sortedWidths);
  ArrayList<Integer> patterns = new ArrayList<>();

  patternEnum (roll_width, sortedWidths, 0, patterns);

  // Initialize and load cutting-stock model from file

  AMPL ampl = new AMPL();

  try {
    ampl.read("cut.mod");
```

Pattern Enumeration in Java

Send data to AMPL

```
ampl.getParameter("overrun").setValues(overrun);
int numPatterns = patterns.size() / widths.length;
ampl.getParameter("nPatterns").setValues(numPatterns);

DataFrame widthOrder = new DataFrame(1, "WIDTHS", "order");
widthOrder.setColumn("WIDTHS", widths);
widthOrder.setColumn("order", orders);
ampl.setData(widthOrder, true);

DataFrame allPatterns = new DataFrame(2, "WIDTHS", "PATTERNS", "rolls");
for (int i = 0; i < widths.length; i++) {
  for (int j = 0; j < numPatterns; j++) {
    allPatterns.addRow(
      sortedWidths[i], j + 1, patterns.get(j * widths.length + i));
  }
}
ampl.setData(allPatterns, false);
```

AMPL API

Pattern Enumeration in Java

Solve and report solution

```
      ampl.setOption("solver", "gurobi");
      ampl.solve();
      printSolution (ampl.getVariable("Cut"), ampl.getParameter("rolls"));
    } finally {
      ampl.close();
    }
  }
```

In practice . . .

Integrate within a larger scheme

Retain benefits of algebraic modeling

- work with natural representation of optimization models
- efficient prototyping, reliable maintenance

Use the best tools for each part of the project

- program data manipulation in your choice of language
- work with optimization models in AMPL

Pattern Generation in MATLAB

Set up AMPL, get data

```
function cuttingGen(dataFile)

% Initialize

ampl = AMPL();

% Load cutting-stock model from file

ampl.read('cut.mod');
Cut = ampl.getVariable('Cut');
Limits = ampl.getConstraint('FinishedRollLimits');

% Get data from .mat file: roll_width, overrun, widths, orders

load(dataFile);
```

AMPL API

Pattern Generation in MATLAB

Send data to AMPL

```
% Send scalar values

ampl.getParameter('overrun').setValues(overrun);
ampl.getParameter('nPatterns').setValues(length(widths));

% Send order vector

WidthOrder = DataFrame(1, 'WIDTHS', 'order');
WidthOrder.setColumn('WIDTHS', num2cell(widths));
WidthOrder.setColumn('order', orders);
ampl.setData(WidthOrder, 'WIDTHS');

% Generate and send initial pattern matrix

maxpat = floor(roll_width./widths);
patmat = diag(maxpat);

InitPatterns = DataFrame(2, 'WIDTHS', 'PATTERNS', 'rolls');
InitPatterns.setMatrix(patmat, num2cell(widths), num2cell(1:length(widths)));
ampl.setData(InitPatterns);
```

AMPL API

Pattern Generation in MATLAB

Set up for generation loop

```
% Set solve options

ampl.setOption('solver','gurobi');
ampl.setOption('relax_integrality','1');

% Set up DataFrame for sending AMPL new patterns

ampl.eval('param newpat {WIDTHS} integer >= 0;');

NewPattern = DataFrame(1, 'WIDTHS', 'newpat');
NewPattern.setColumn('WIDTHS', num2cell(widths));

% Compute multiplier for integer weights

[n,d] = rat(widths);
intmult = lcms(d);
```

Pattern Generation in MATLAB

Loop 1: Retrieve duals & look for new pattern

```
while 1

   ampl.solve

   DualPrices = Limits.getValues;
   dualvec = DualPrices.getColumnAsDoubles('dual');

   wgt = []; val = [];
   for w = 1:length(widths)
      if dualvec(w) > 0
         wgt = [wgt widths(w)*ones(1,maxpat(w))];
         val = [val dualvec(w)*ones(1,maxpat(w))];
      end
   end

   % Solve knapsack problem for potential new pattern

   [kmax,z] = kp01 (round(intmult*wgt), val, intmult*roll_width);

   if kmax < 1.000001
     break;
   end
```

AMPL API

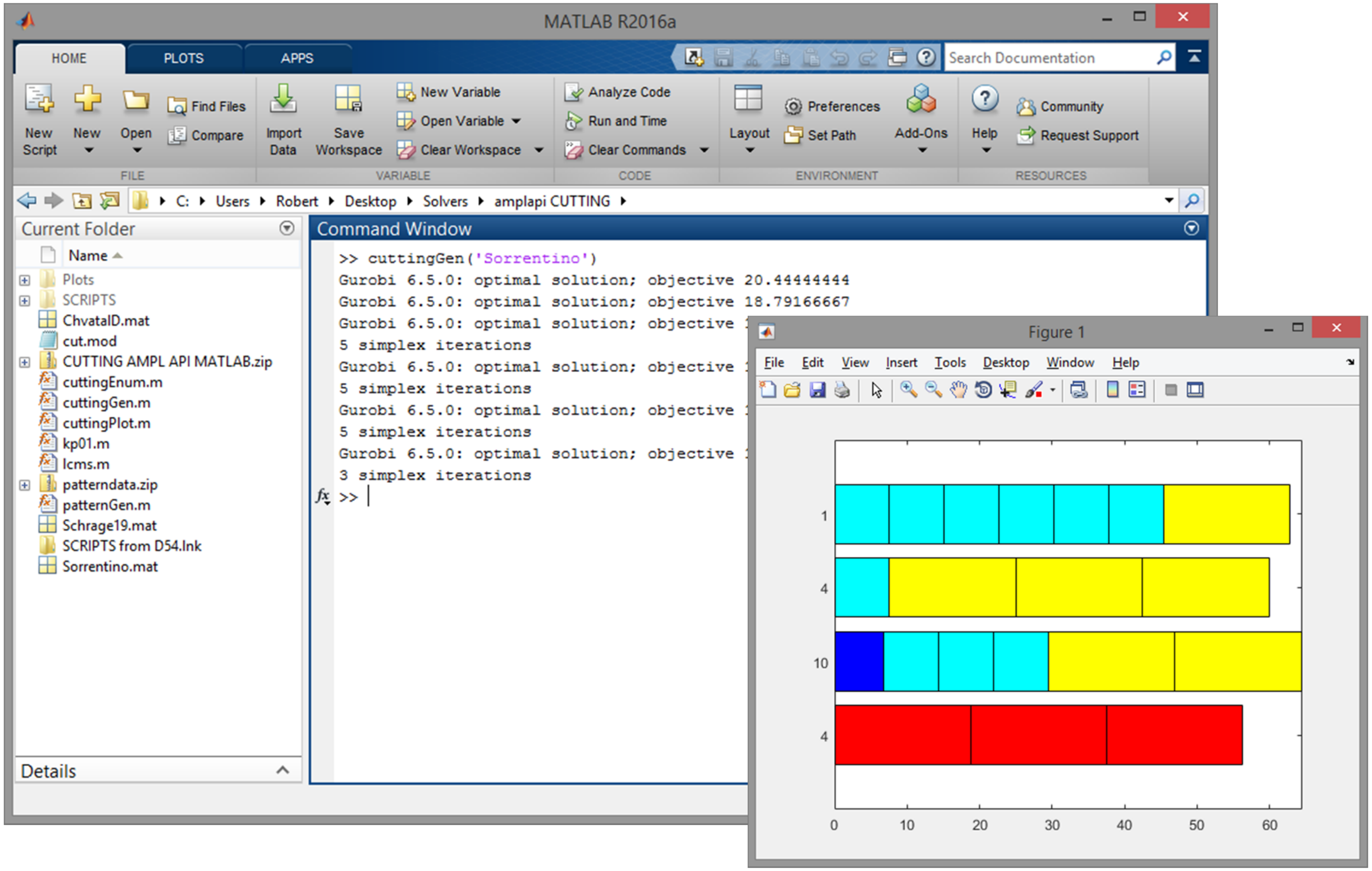
Pattern Generation in MATLAB

Loop 2: Send new pattern to AMPL

```
   widthlist = wgt(z);
   for w = 1:length(widths)
      newpat(w) = length(find(widthlist==widths(w)));
   end
   patmat = [patmat; newpat];

   NewPattern.setColumn('newpat', newpat);
   ampl.setData(NewPattern);

   ampl.eval('let nPatterns := nPatterns + 1;');
   ampl.eval('let {w in WIDTHS} rolls[w,nPatterns] := newpat[w];');
end

% Compute and display integer solution

ampl.setOption('relax_integrality','0');
ampl.solve;

CuttingPlan = Cut.getValues();
cutvec = CuttingPlan.getColumnAsDoubles('val');

cuttingPlot (roll_width, widths, patmat(cutvec>0,:), cutvec(cutvec>0))
```

AMPL API

Pattern Generation in MATLAB



```
>> cuttingGen('Sorrentino')
Gurobi 6.5.0: optimal solution; objective 20.44444444
Gurobi 6.5.0: optimal solution; objective 18.79166667
Gurobi 6.5.0: optimal solution; objective 1
5 simplex iterations
Gurobi 6.5.0: optimal solution; objective 1
5 simplex iterations
Gurobi 6.5.0: optimal solution; objective 1
5 simplex iterations
Gurobi 6.5.0: optimal solution; objective 1
3 simplex iterations
>> |
```

In practice . . .

Implement hybrid iterative schemes

- ❖ build powerful software for hard problems

Alternate between optimization & other analytics

- ❖ invoke specialized optimizers for subproblems

Data Transfer: Alternatives

Process

- ❖ Define symbolic sets & parameters in AMPL model
- ❖ Create corresponding objects in program
- ❖ Transfer data using API methods
  - ∗ Program to AMPL
  - ∗ AMPL to program

Methods for transfer between . . .

- ❖ Scalar values
- ❖ Collections of values
  - ∗ AMPL indexed expressions
  - ∗ Java arrays, MATLAB matrices
- ❖ Relational tables
  - ∗ AMPL “table” structures
  - ∗ API DataFrame objects in Java, MATLAB

Availability

AMPL API version 1.2 released

- Java, MATLAB, C++
- Add-ons to all AMPL distributions
- Download from www.ampl.com/products/api/

More languages to follow

- R
- Python
- .NET: C#, Visual Basic

Try AMPL & Solvers . . .

Freely downloadable small-problem demo

- http://ampl.com/try-ampl/download-a-free-demo/

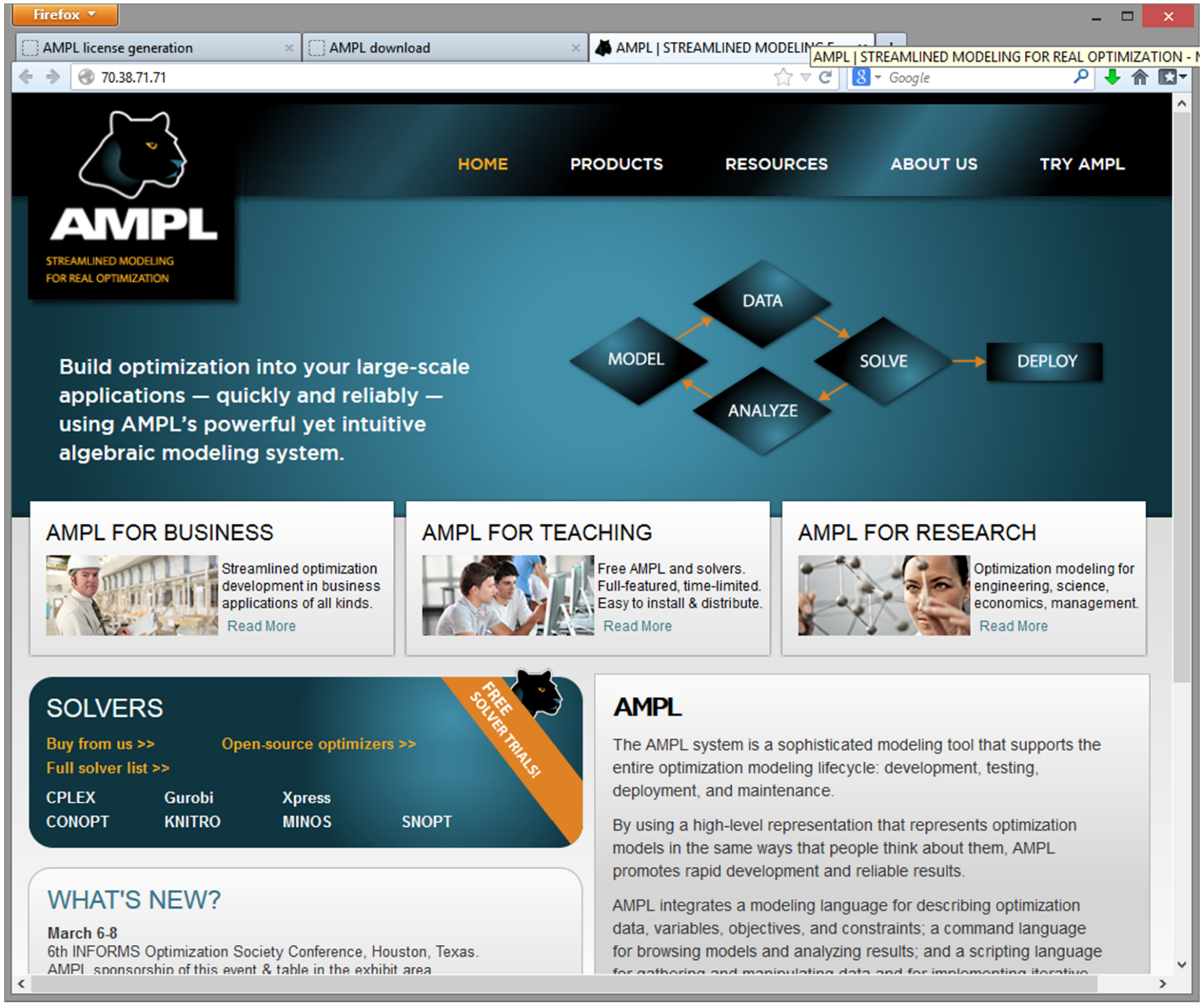
Free submissions to online NEOS Server

- http://neos-server.org/

30-day full trial

- http://ampl.com/try-ampl/request-a-full-trial/

www.ampl.com



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CONOPT KNITRO MINOS SNOPT

WHAT'S NEW?
March 6-8
6th INFORMS Optimization Society Conference, Houston, Texas.
AMPL sponsorship of this event & table in the exhibit area

AMPL
The AMPL system is a sophisticated modeling tool that supports the entire optimization modeling lifecycle: development, testing, deployment, and maintenance.

By using a high-level representation that represents optimization models in the same ways that people think about them, AMPL promotes rapid development and reliable results.

AMPL integrates a modeling language for describing optimization data, variables, objectives, and constraints; a command language for browsing models and analyzing results; and a scripting language

AMPL Readings

- R. Fourer, "Modeling Languages versus Matrix Generators for Linear Programming." *ACM Transactions on Mathematical Software* **9** (1983) 143–183.
- R. Fourer, D.M. Gay, B.W. Kernighan, "A Modeling Language for Mathematical Programming." *Management Science* **36** (1990) 519–554.
- Robert Fourer, "Database Structures for Mathematical Programming Models." *Decision Support Systems* **20** (1997) 317–344.
- R. Fourer, D.M. Gay, B.W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*. Duxbury Press, Belmont, CA (first edition 1993, second edition 2003).
- Robert Fourer, On the Evolution of Optimization Modeling Systems. M. Groetschel (ed.), *Optimization Stories*. Documenta Mathematica (2012) 377-388.