New Python Integration Features of the AMPL Modeling Language

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New Python Integration Features of the AMPL Modeling Language

Optimization modeling languages are fundamentally declarative in design, but are frequently put to use within broader contexts that require a variety of programming options. Thus while programming is not employed to describe models, it facilitates the integration of models into broader algorithmic schemes and business applications. This presentation focuses on integration of the widely used AMPL modeling language with Python and Jupyter, the most popular environment for programming in data science. A single running example illustrates multiple topics, which include integrating model-based optimization into applications using AMPL's Python API, embedding Python in AMPL models and scripts, implementing complex AMPL constraint generators in Python, and setting up solver callbacks using Python programs.

Examples

AMPL Python API

- Example: Roll cutting by pattern enumeration

Python data embedded in an AMPL model

- Example: Roll cutting data

Python code embedded in an AMPL model

- Example: Generating advanced lot-sizing constraints

Python callbacks from Gurobi

- Example: User-specified stopping rule

AMPL in Jupyter notebooks

- Example: Roll cutting by pattern generation
- Example: Lot sizing using advanced formulations

AMPL Python API

Example: ***Roll Cutting by Pattern Enumeration***

- Fill orders for rolls of various widths

Given

- Raw rolls of a large (fixed) width
- Demands for various (smaller) ordered widths
- Selected cutting patterns that may be used

Determine

- Number of times to cut each pattern

So that

- Demands are met (or slightly exceeded)
- *Number of raw rolls cut* is minimized

Mathematical Formulation

Given

- w width of "raw" rolls
- W set of (smaller) ordered widths
- n number of cutting patterns considered

and

- a_{ij} occurrences of width i in pattern j, for each $i \in W$ and $j = 1, \ldots, n$
- b_i orders for width i, for each $i \in W$
- o limit on overruns

Mathematical Formulation *(cont'd)*

Determine

X_j number of rolls to cut using pattern j,
for each $j = 1, \ldots, n$

to minimize

$\sum_{j=1}^{n} X_j$

total number of rolls cut

subject to

$b_i \leq \sum_{j=1}^{n} a_{ij} X_j \leq b_i + o$, for all $i \in W$

number of rolls of width i cut
must be at least the number ordered,
and must be within the overrun limit

AMPL Formulation

Symbolic model

```
param rawWidth;
set WIDTHS;

param nPatterns integer > 0;
set PATTERNS = 1..nPatterns;

param rolls {WIDTHS,PATTERNS} >= 0, default 0;

param order {WIDTHS} >= 0;
param overrun;

var Cut {PATTERNS} integer >= 0;

minimize TotalCut: sum {p in PATTERNS} Cut[p];

subject to OrderLimits {w in WIDTHS}:
  order[w] <= sum {p in PATTERNS} rolls[w,p] * Cut[p] <= order[w] + overrun;
```

$$b_i \leq \sum_{j=1}^{n} a_{ij} X_j \leq b_i + o$$

AMPL Formulation *(cont'd)*

Explicit data (independent of model)

```
param rawWidth := 64.5 ;

param: WIDTHS: order :=
         6.77    10
         7.56    40
        17.46    33
        18.76    10 ;

param nPatterns := 9 ;

param rolls:  1  2  3  4  5  6  7  8  9 :=
       6.77   0  1  1  0  3  2  0  1  4
       7.56   1  0  2  1  1  4  6  5  2
      17.46   0  1  0  2  1  0  1  1  1
      18.76   3  2  2  1  1  1  0  0  0 ;

param overrun := 6 ;
```

Command Language *(cont'd)*

Solver choice independent of model and data

```
ampl: model cut.mod;
ampl: data cut.dat;

ampl: option solver gurobi;

ampl: solve;

Gurobi 8.1.0: optimal solution; objective 20
3 simplex iterations
1 branch-and-cut nodes

ampl: option omit_zero_rows 1;
ampl: option display_1col 0;

ampl: display Cut;

4 13   7 4   9 3
```

AMPL Command Language

Model + data = problem instance to be solved

```
ampl: model cut.mod;
ampl: data cut.dat;

ampl: option solver cplex;

ampl: solve;

CPLEX 12.9.0.0: optimal integer solution; objective 20
3 MIP simplex iterations
0 branch-and-bound nodes

ampl: option omit_zero_rows 1;
ampl: option display_1col 0;

ampl: display Cut;

4 13   7 4   9 3
```

AMPL Command Language

Model + data = problem instance to be solved

```
ampl: model cut.mod;
ampl: data cut.dat;

ampl: option solver xpress;

ampl: solve;

XPRESS 8.5(32.01.08): Global search complete
Best integer solution found 20
3 integer solutions have been found
1 branch and bound node

ampl: option omit_zero_rows 1;
ampl: option display_1col 0;

ampl: display Cut;

4 13   7 4   9 3
```

AMPL Model

Command Language *(cont'd)*

Results available for browsing

```
ampl: display {p in PATTERNS} sum {w in WIDTHS} w * rolls[w,p];

1 63.84   3 59.41   5 64.09   7 62.82   9 59.66   # material used
2 61.75   4 61.24   6 62.54   8 62.0              # in each pattern

ampl: display sum {p in PATTERNS}
ampl?   Cut[p] * (rawWidth - sum {w in WIDTHS} w * rolls[w,p]);

62.32                                             # total waste
                                                  # in solution

ampl: display OrderLimits.lslack;

 6.77  0                                          # overruns
 7.56  0                                          # of each pattern
17.46  0
18.76  5
```

AMPL APIs

Principles

- APIs for “all” popular languages
  - C++, C#, Java, MATLAB, Python, R
- Common overall design
- Common implementation core in C++
- Customizations for each language and its data structures

Python support: amplpy

- Versions: 2.7, 3.3 and up
- Data structures: Lists, dictionaries, dataframes
- Libraries: Pandas, Bokeh
- Easy installation: *pip install amplpy*

Roll Cutting by Pattern Enumeration

Principles

- Generate a long list of candidate patterns
  - for this example, all nondominated patterns
- Solve the cutting problem using this entire candidate list

Implementation

- Pattern enumeration in Python
- Modeling and solving in AMPL, via API calls
- Solution reporting in Python

Key to examples

- AMPL entities
- AMPL API Python objects
- AMPL API Python methods
- Python functions etc.

AMPL Model File

Same pattern-cutting model

```
param nPatterns integer > 0;

set PATTERNS = 1..nPatterns;  # patterns
set WIDTHS;                   # finished widths

param order {WIDTHS} >= 0;    # rolls of width j ordered
param overrun;                # permitted overrun on any width

param rawWidth;                    # width of raw rolls to be cut
param rolls {WIDTHS,PATTERNS} >= 0, default 0;
                                   # rolls of width i in pattern j

var Cut {PATTERNS} integer >= 0;   # raw rolls to cut in each pattern

minimize TotalRawRolls: sum {p in PATTERNS} Cut[p];

subject to FinishedRollLimits {w in WIDTHS}:
   order[w] <= sum {p in PATTERNS} rolls[w,p] * Cut[p] <= order[w] + overrun;
```

Some Python Data

A float, an integer, and a dictionary

```
roll_width = 64.5

overrun = 6

Orders = {
     6.77: 10,
     7.56: 40,
    17.46: 33,
    18.76: 10
}
```

. . . can also work with lists and Pandas dataframes

Pattern Enumeration

Load & generate data, set up AMPL model

```
def cuttingEnum(dataset):
  from amplpy import AMPL

  # Read orders, roll_width, overrun

  exec(open(dataset+'.py').read(), globals())

  # Enumerate patterns

  widths = list(sorted(orders.keys(), reverse=True))
  patmat = patternEnum(roll_width, widths)

  # Set up model

  ampl = AMPL()
  ampl.option['ampl_include'] = 'models'
  ampl.read('cut.mod')
```

Pattern Enumeration

Send data to AMPL

```
# Send scalar values

ampl.param['nPatterns'] = len(patmat)
ampl.param['overrun'] = overrun
ampl.param['rawWidth'] = roll_width

# Send order vector

ampl.set['WIDTHS'] = widths
ampl.param['order'] = orders

# Send pattern matrix

ampl.param['rolls'] = {
   (widths[i], 1+p): patmat[p][i]
   for i in range(len(widths))
   for p in range(len(patmat))
}
```

Pattern Enumeration

Solve and get results

```
# Solve

ampl.option['solver'] = 'gurobi'
ampl.solve()

# Retrieve solution

CuttingPlan = ampl.var['Cut'].getValues()
cutvec = list(CuttingPlan.getColumn('Cut.val'))
```

AMPL Python API

Pattern Enumeration

Display solution

```
# Prepare solution data

summary = {
    'Data': dataset,
    'Obj': int(ampl.obj['TotalRawRolls'].value()),
    'Waste': ampl.getValue(
                 'sum {p in PATTERNS} Cut[p] * \
                    (rawWidth - sum {w in WIDTHS} w*rolls[w,p])'
             )
}

solution = [
    (patmat[p], cutvec[p])
    for p in range(len(patmat))
    if cutvec[p] > 0
]


# Create plot of solution

cuttingPlot(roll_width, widths, summary, solution)
```

Pattern Enumeration

Enumeration routine

```
def patternEnum(roll_width, widths, prefix=[]):
  from math import floor

  max_rep = int(floor(roll_width/widths[0]))

  if len(widths) == 1:
      patmat = [prefix+[max_rep]]

  else:
     patmat = []
     for n in reversed(range(max_rep+1)):
        patmat += patternEnum(roll_width-n*widths[0], widths[1:], prefix+[n])

  return patmat
```

AMPL Python API

Pattern Enumeration

Plotting routine

```
def cuttingPlot(roll_width, widths, summ, solution):
    import numpy as np
    import matplotlib.pyplot as plt

    ind = np.arange(len(solution))
    acc = [0]*len(solution)

    colorlist = ['red','lightblue','orange','lightgreen',
                 'brown','fuchsia','silver','goldenrod']
```

Pattern Enumeration

Plotting routine (cont'd)

```
for p, (patt, rep) in enumerate(solution):
   for i in range(len(widths)):
      for j in range(patt[i]):
         vec = [0]*len(solution)
         vec[p] = widths[i]
         plt.barh(ind, vec, 0.6, acc,
                   color=colorlist[i%len(colorlist)], edgecolor='black')
         acc[p] += widths[i]

plt.title(summ['Data'] + ": " +
   str(summ['Obj']) + " rolls" + ", " +
   str(round(100*summ['Waste']/(roll_width*summ['Obj']),2)) + "% waste"
   )

plt.xlim(0, roll_width)
plt.xticks(np.arange(0, roll_width, 10))
plt.yticks(ind, tuple("x {:}".format(rep) for patt, rep in solution))

plt.show()
```

Pattern Enumeration

Robert: running ipython

File Edit Help

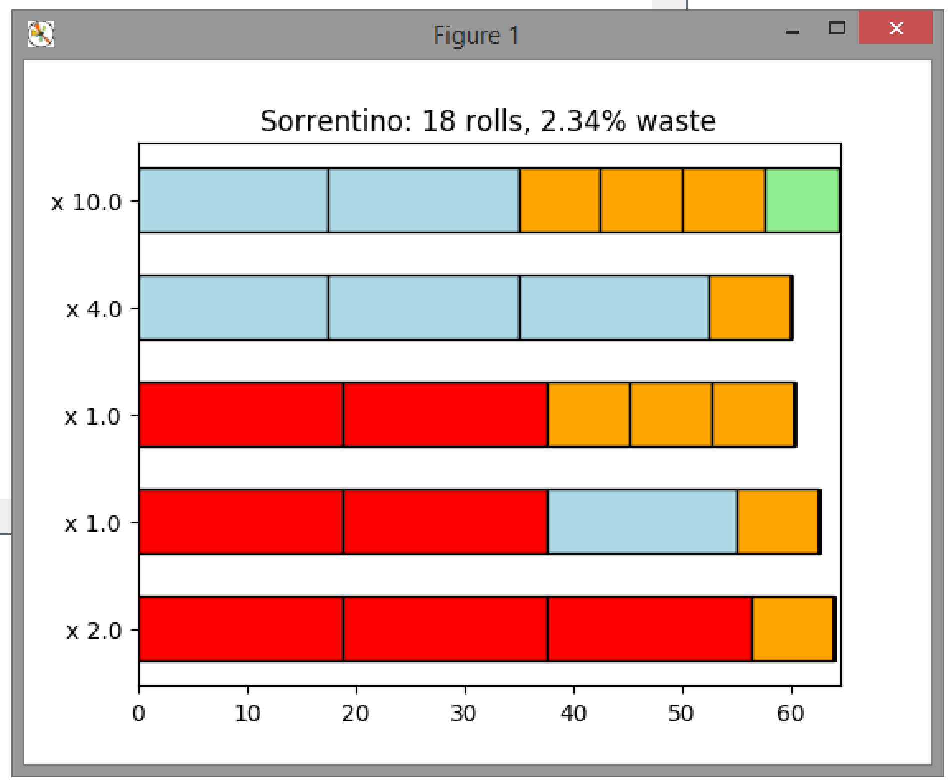
```
sw: ipython
Python 3.6.4 (v3.6.4:d48eceb, Dec 19 2017, 06:54:40) [MSC v.1900 64 bit (AMD64)]
Type 'copyright', 'credits' or 'license' for more information
IPython 6.2.1 -- An enhanced Interactive Python. Type '?' for help.

In [1]: from pattern_enumeration import *

In [2]: cuttingEnum('Sorrentino')

43 patterns enumerated

Gurobi 8.1.0: optimal solution; objective 18
7 simplex iterations
1 branch-and-cut nodes
```



Python Data Embedded in an AMPL Model

Example: Roll Cutting Data

Data transfer method approach (amplpy)

- Read the model into AMPL
- Use Python API methods to send data to AMPL

Embedded data approach (PyMPL)

- Specify Python data correspondences in the model
- Read the model into AMPL

Getting Data

Imported and generated data in Python

```
roll_width = 64.5

overrun = 6

orders = {
    6.77: 10,
    7.56: 40,
    17.46: 33,
    18.76: 10
}

patmat = patternEnum(roll_width, list(sorted(orders.keys(), reverse=True)))
```

Python Integration

Sending Data using the Python API

Specify symbolic sets and parameters in AMPL

cut.mod

```
param nPatterns integer > 0;

set PATTERNS = 1..nPatterns;
set WIDTHS;

param order {WIDTHS} >= 0;
param overrun;

param rawWidth;
param rolls {WIDTHS,PATTERNS} >= 0, default 0;
```

Sending Data using the Python API *(cont'd)*

Call `ampl` *methods to read model, send data*

```
ampl = AMPL()
ampl.read('cut.mod')

.......

ampl.param['nPatterns'] = len(patmat)
ampl.param['overrun'] = overrun
ampl.param['rawWidth'] = roll_width

ampl.set['WIDTHS'] = widths
ampl.param['order'] = orders

ampl.param['rolls'] = {
    (widths[i], 1+p): patmat[p][i]
    for i in range(len(widths))
    for p in range(len(patmat))
}
```

Sending Data using PyMPL

Specify Python data correspondences in the model

cutpy.mod

```
$PARAM[nPatterns]{ len(patmat) };
set PATTERNS = 1..nPatterns;

$SET[WIDTHS]{ widths };

$PARAM[order{^WIDTHS}]{ orders };
$PARAM[overrun]{ overrun };

$PARAM[rawWidth]{ roll_width };
$PARAM[rolls{^WIDTHS,^PATTERNS}]{
    {
        (widths[i], 1+p): patmat[p][i]
        for i in range(len(widths))
        for p in range(len(patmat))
    }
};
```

```
ampl = AMPL(langext=PyMPL())
ampl.read('cutpy.mod')
```

Python Code Embedded in an AMPL Model

Example: ***Generating advanced lot-sizing constraints***

- Create a tighter formulation that solves faster

New constructs for embedding Python in AMPL

- **$** *python-expression* **$**
- **$EXEC{** *python-statements* **}**

Executing Python inside AMPL

Fix AMPL variables according to Python variable

lotsize.mod

```
$PARAM[NT]{8};

var x {1..NT}, >= 0;   # production lot size
var y {1..NT}, binary; # production set-up
var s {0..NT}, >= 0;   # inventory level

var r {1..NT}, ${">= 0" if BACKLOG else ">= 0, <= 0"}$;

                                      # use these variables iff BACKLOG > 0
```

Executing Python inside AMPL

Invoke Python generators for special lot-sizing constraints

lotsize.mod

```
$EXEC{
def mrange(a, b):
    return range(a, b+1)
s = ['s[{}]'.format(t) for t in mrange(0, NT)]
y = ['y[{}]'.format(t) for t in mrange(1, NT)]
d = [demand[t] for t in mrange(1, NT)]
if BACKLOG is False:
    WW_U_AMPL(s, y, d, NT, prefix='w')
else:
    r = ['r[{}]'.format(t) for t in mrange(1, NT)]
    WW_U_B_AMPL(s, r, y, d, NT, prefix='w')
};
```

```
ampl = AMPL(langext=PyMPL())
ampl.read('lotsize.mod')
ampl.solve()
```

Executing Python inside AMPL

Optional listing of generated constraints

```
var ws {wi in 0..8} = s[wi];
var wr {wi in 1..8} = r[wi];
var wy {wi in 1..8} = y[wi];

param wD {1..8, 1..8};

data;

param wD :=
[1,1]400 [1,2]800 [1,3]1600 [1,4]2400 [1,5]3600 [1,6]4800 [1,7]6000 [1,8]7200
[2,1]0   [2,2]400 [2,3]1200 [2,4]2000 [2,5]3200 [2,6]4400 [2,7]5600 [2,8]6800
[3,1]0   [3,2]0   [3,3]800  [3,4]1600 [3,5]2800 [3,6]4000 [3,7]5200 [3,8]6400
[4,1]0   [4,2]0   [4,3]0    [4,4]800  [4,5]2000 [4,6]3200 [4,7]4400 [4,8]5600
[5,1]0   [5,2]0   [5,3]0    [5,4]0    [5,5]1200 [5,6]2400 [5,7]3600 [5,8]4800
[6,1]0   [6,2]0   [6,3]0    [6,4]0    [6,5]0    [6,6]1200 [6,7]2400 [6,8]3600
[7,1]0   [7,2]0   [7,3]0    [7,4]0    [7,5]0    [7,6]0    [7,7]1200 [7,8]2400
[8,1]0   [8,2]0   [8,3]0    [8,4]0    [8,5]0    [8,6]0    [8,7]0    [8,8]1200
;

model;
```

Python Integration

Executing Python inside AMPL

Optional listing of generated constraints (cont'd)

```
var wa {1..8};
var wb {1..8};

subject to wXY {wt in 1..8}: wa[wt] + wb[wt] + wy[wt] >= 1;

subject to wXA {wk in 1..8, wt in wk..min(8, wk+8-1): wD[wt,wt]>0}:

    ws[wk-1] >=
        sum {wi in wk..wt} wD[wi,wi] * wa[wi]
        - sum {wi in wk..wt-1} wD[wi+1,wt] * wy[wi];

subject to wXB {wk in 1..8, wt in max(1, wk-8+1)..wk: wD[wt,wt]>0}:

    wr[wk] >=
        sum {wi in wt..wk} wD[wi,wi] * wb[wi]
        - sum {wi in wt+1..wk} wD[wt,wi-1] * wy[wi];
```

Python Callbacks from Gurobi

Example: ***User-Specified Stopping Rule***

Data

- Times $t_1 < \mathrm{t}_2 < \mathrm{t}_3$ etc.
- Optimality gap tolerances $g_1 < g_2 < g_3$ etc.

Execution

- When elapsed time reaches t_i . . .
- Increase the gap tolerance to g_i

Callbacks

Stopping rule data in Python dictionary

stopping.py

```
stopdict = { 'time'   : (  15,    30,   60 ),
             'gaptol' : ( .0002, .002, .02 )
           }
```

Main routine for cutting by pattern generation

pattern_generation.py

```
def cuttingGen(cutdata, stopdata = ""):
  from amplpy import AMPL
  .......
  # begin pattern generation phase
  # finish when continuous relaxation of cutting problem has been solved
```

Python Integration

Callbacks

Set up callback and solve final integer program

```
# Instead of Master.solve(), export to a gurobipy object
grb_model = Master.exportGurobiModel()

# Assign AMPL stopping data to gurobipy objects
if len(stopdata) == 0:
  grb_model._stoprule = {'time': (1e+10,), 'gaptol': (1,)}
else:
  exec(open(stopdata+'.py').read(), globals())
  stopdict['time'] += (1e+10,)
  stopdict['gaptol'] += (1,)
  grb_model._stoprule = stopdict
grb_model._current = 0

# Solve and import results
grb_model.optimize(callback)
Master.importGurobiSolution(grb_model)
```

Python Integration

Callbacks

Callback function

```
def callback(m, where):
  """Gurobi callback function."""
  if where == gpy.GRB.Callback.MIP:
    runtime = m.cbGet(gpy.GRB.Callback.RUNTIME)
    if runtime >= m._stoprule['time'][m._current]:
      print("Reducing gap tolerance to %f at %d seconds" % \
        (m._stoprule['gaptol'][m._current], m._stoprule['time'][m._current]))
      m.Params.MIPGap = m._stoprule['gaptol'][m._current]
      m._current += 1
```

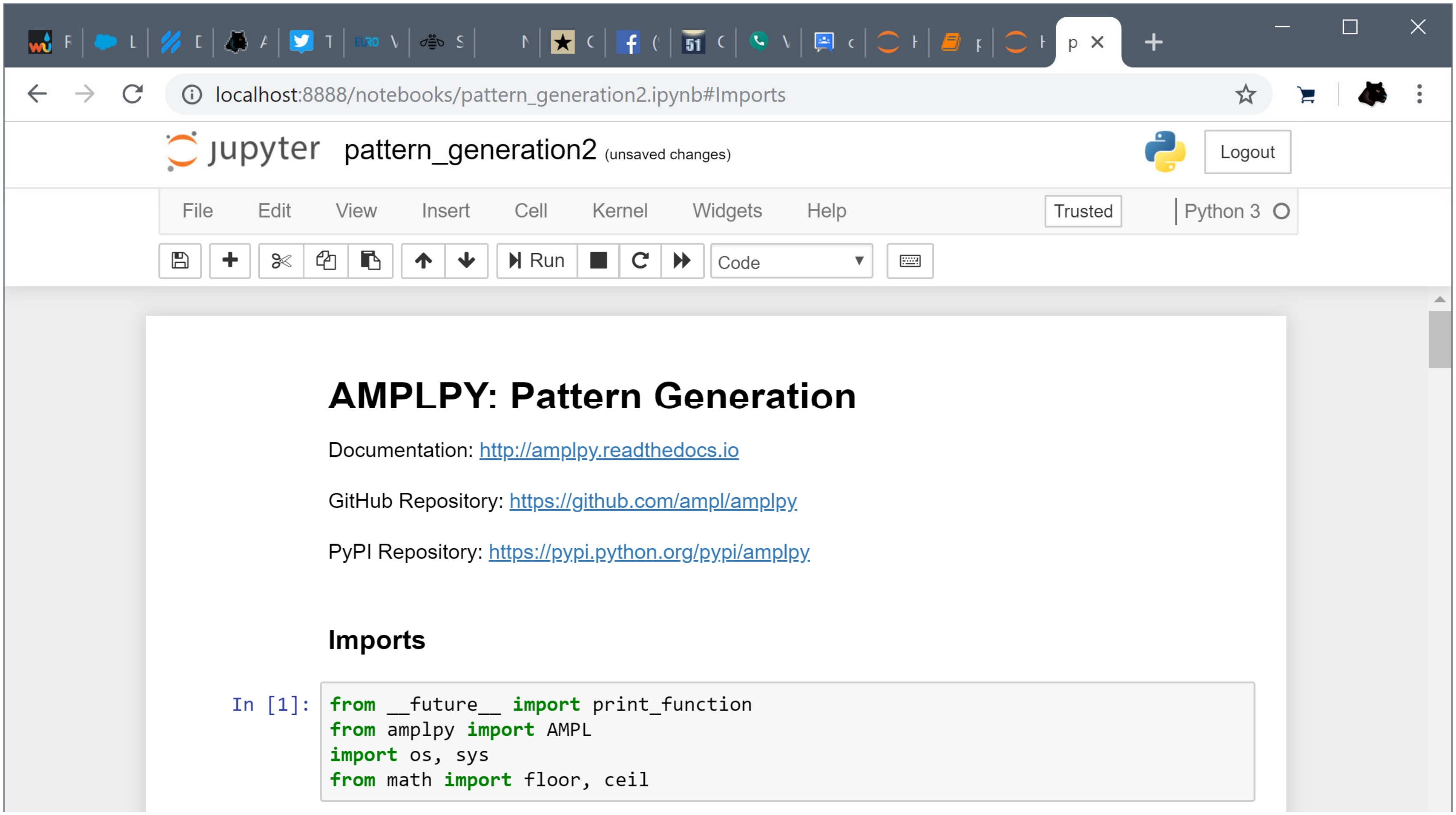
AMPL in Jupyter Notebooks

Example: Roll Cutting by Pattern Generation

Example: Lot Sizing Using Advanced Formulations

AMPL in Jupyter Notebooks

Contact the speaker (4er@ampl.com) for the notebook files



Status

AMPL API 2.0

- Released

Embedded Python data & code
Python callbacks from Gurobi
AMPL in Jupyter notebooks

- All available for beta testing
- Contact support@ampl.com for more information