

INFORMS Annual Meeting 2025 | Technology Workshop

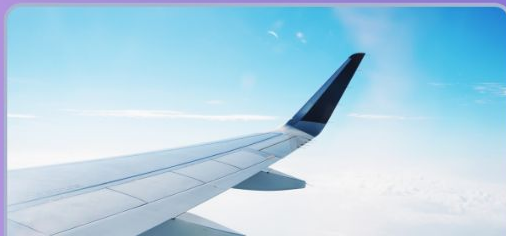


From Classroom to Industry

Modern Optimization with AMPL
for Energy, Finance, Supply
Chain and Beyond

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Outline

1. Modern Optimization
 - How Optimization generates millions?
 - Industry level experience in Academia
 - Writing code in 2025
 - Amplbot
2. Hands-on Amplpy → ampl.com/workshop
3. AMPL/MP: Complex models can be made easy
 - Automatic Reformulations & Enhanced solvers



What is AMPL?

Mathematical Optimization Engine

→ Express problems as you think about them

- Large-scale modeling
- Linear, Non-Linear, Constraint Programming...

→ Solver interfaces

- Open-source: HiGHS, Scip, CBC, ipopt, . . .
- Commercial: CPLEX, Gurobi, Xpress, Knitro, COPT . . .

→ Amplpy

→ Other APIs

- C++, C#, Java, MATLAB, R

Models aren't just “min cx s.t. $Ax=b$ ”

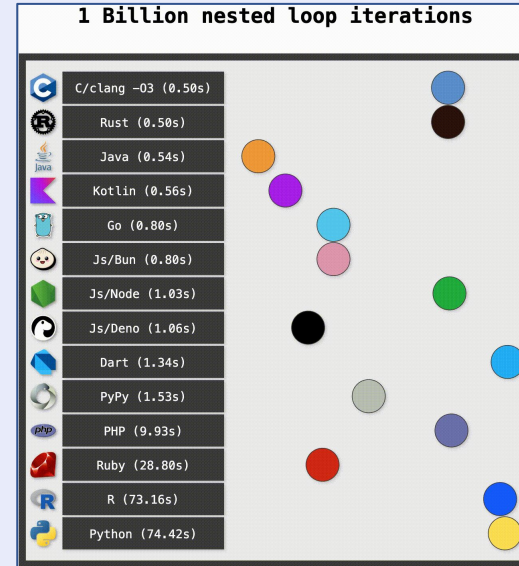
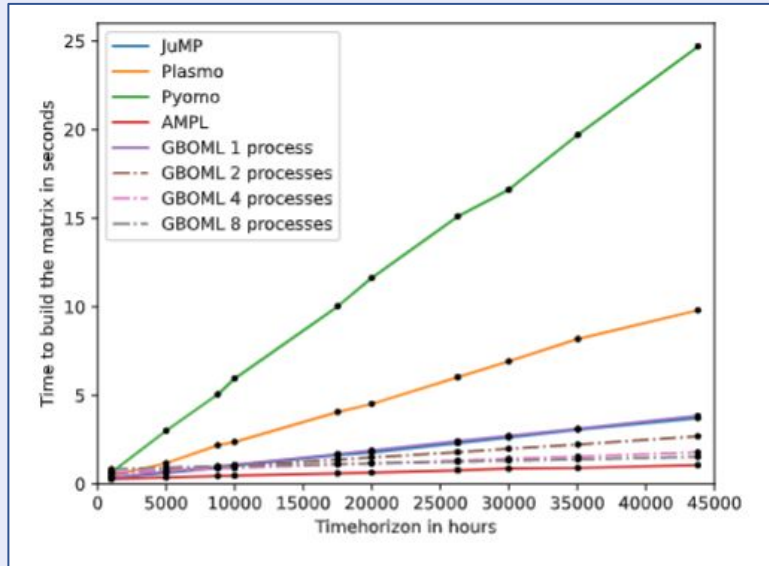
Example: If you are charging a thermal battery, you need to charge at a stable rate:

```
set Times ordered;
param MaxChargeRate;

var ChargeRate{t in Times} >= 0 <= MaxChargeRate;

subject to ChargeRateVariationLimit{t in Times: ord(t) >= 2}:
    ChargeRate[t] > 0 && ChargeRate[prev(t)] > 0
    ==>
    abs(ChargeRate[t]-ChargeRate[prev(t)]) <= 10;
```

Third-party benchmarks



Ref: GBOML: A Structure-Exploiting Optimization Modelling Language in Python
Bardhyl Miftari et al. (2023)

GBOML: a structure-exploiting optimization modelling language in Python

Bardhyl Miftari ✉, Mathias Berger, Guillaume Derval, Quentin Louveaux & Damien Ernst

Pages 227-256 | Received 30 Nov 2022, Accepted 06 Aug 2023, Published online: 08 Sep 2023

🗨️ Cite this article 🔗 <https://doi.org/10.1080/10556788.2023.2246169>



📄 Full Article

📊 Figures & data

📖 References

🗨️ Citations

📈 Metrics

📄 Reprints & Permissions

Read this article

Abstract

Mixed-Integer Linear Programs (MILPs) have many practical applications. Most modelling tools for MILPs fall in two broad categories. Indeed, tools such as algebraic modelling languages allow practitioners to compactly encode models using syntax close to mathematical notation but usually lack support for special structures, while other tools instead provide predefined components that can be easily assembled but modifying or adding new components is difficult. In this work, we present the inner workings of the Graph-Based Optimization Modelling Language (GBOML), an open-source modelling tool implemented in Python combining the strengths of both worlds. GBOML natively supports special structures that can be encoded by a hierarchical hypergraph, offers syntax close to mathematical notation and facilitates the modular construction and reuse of time-indexed models. We detail design choices enabling these features and show that they simplify problem encoding, lead to faster instance generation times and sometimes faster solve times. We benchmark the times taken by GBOML, JuMP, PlasmO, Pyomo and AMPL to generate instances of a structured MILP. We find that GBOML outperforms PlasmO and Pyomo, is tied with JuMP but is slower than AMPL. With parallel model generation, GBOML outperforms JuMP and closes the gap with AMPL. GBOML has the smallest memory footprint.

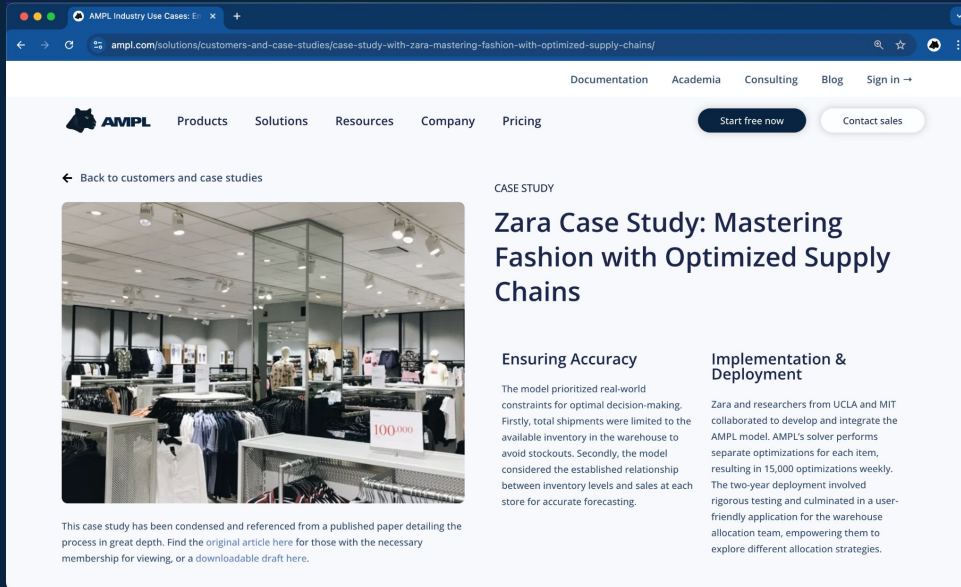
Where is AMPL used most?

- Large scale optimization applications where highly efficient optimization software is essentially indispensable due to **performance requirements**.
- Very **complex and detailed models** incorporating complex **business details** and **regulations**.
- Mission-critical applications where mistakes can be very expensive (e.g., damaging equipment) and **transparent modeling is essential**.
- Competitive environments where new models must be **quick to develop and deploy**.

Part I

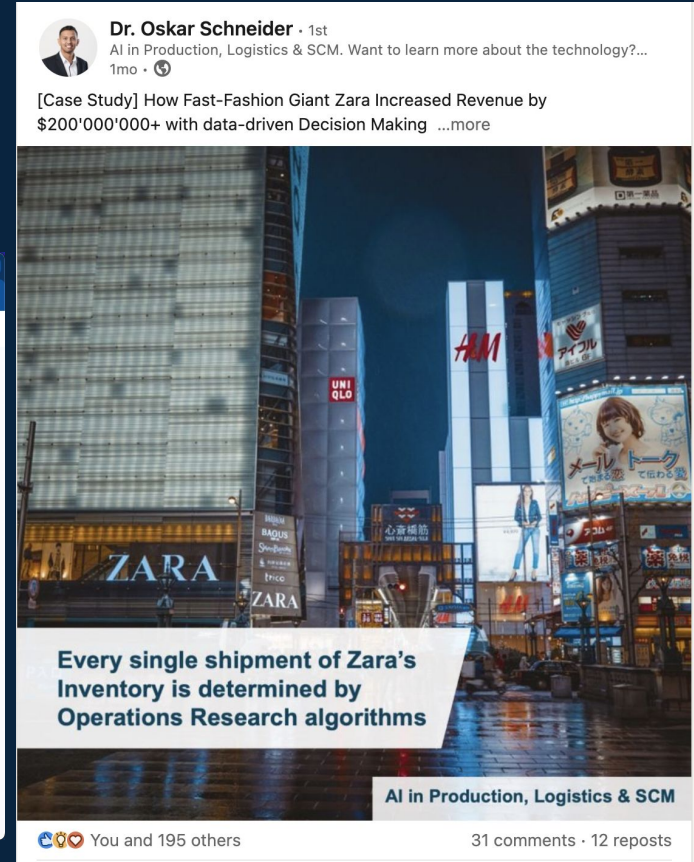
How Optimization Generates Millions

Zara Case Study



The screenshot shows the AMPL website with the URL ampl.com/solutions/customers-and-case-studies/case-study-with-zara-mastering-fashion-with-optimized-supply-chains/. The page features a navigation bar with links to Documentation, Academia, Consulting, Blog, and Sign in. Below the navigation bar, there is a section titled "CASE STUDY" with the heading "Zara Case Study: Mastering Fashion with Optimized Supply Chains". The page includes an image of a Zara store interior and a text block describing the case study. The text mentions that the model prioritized real-world constraints for optimal decision-making, such as limiting total shipments to the available inventory in the warehouse to avoid stockouts. It also states that the model considered the established relationship between inventory levels and sales at each store for accurate forecasting. The implementation involved collaboration with researchers from UCLA and MIT to develop and integrate the AMPL model, which performs separate optimizations for each item, resulting in 15,000 optimizations weekly. The two-year deployment involved rigorous testing and culminated in a user-friendly application for the warehouse allocation team, empowering them to explore different allocation strategies.

AMPL Industry Use Cases: E...
ampl.com/solutions/customers-and-case-studies/case-study-with-zara-mastering-fashion-with-optimized-supply-chains/
Documentation Academia Consulting Blog Sign in →
AMPL Products Solutions Resources Company Pricing
Start free now Contact sales
← Back to customers and case studies
CASE STUDY
Zara Case Study: Mastering Fashion with Optimized Supply Chains
Ensuring Accuracy
The model prioritized real-world constraints for optimal decision-making. Firstly, total shipments were limited to the available inventory in the warehouse to avoid stockouts. Secondly, the model considered the established relationship between inventory levels and sales at each store for accurate forecasting.
Implementation & Deployment
Zara and researchers from UCLA and MIT collaborated to develop and integrate the AMPL model. AMPL's solver performs separate optimizations for each item, resulting in 15,000 optimizations weekly. The two-year deployment involved rigorous testing and culminated in a user-friendly application for the warehouse allocation team, empowering them to explore different allocation strategies.
This case study has been condensed and referenced from a published paper detailing the process in great depth. Find the original article [here](#) for those with the necessary membership for viewing, or a downloadable draft [here](#).

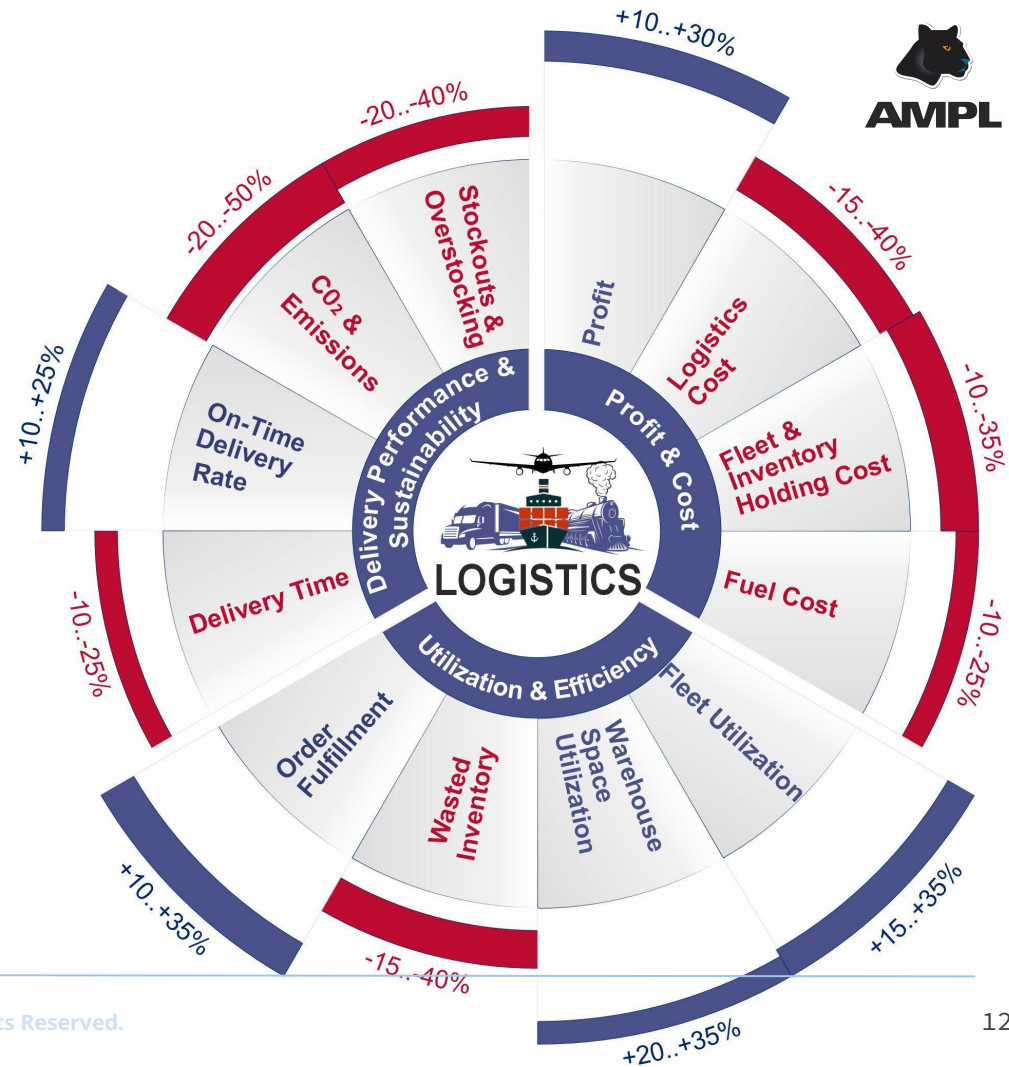


The social media post is by Dr. Oskar Schneider, 1st, with a bio: "AI in Production, Logistics & SCM. Want to learn more about the technology?... 1mo". The post title is "[Case Study] How Fast-Fashion Giant Zara Increased Revenue by \$200'000'000+ with data-driven Decision Making ...more". The image shows a night view of a city street with a Zara store and other buildings. A text overlay on the image reads: "Every single shipment of Zara's Inventory is determined by Operations Research algorithms". Below the image, it says "AI in Production, Logistics & SCM". The post has 195 likes and 31 comments.

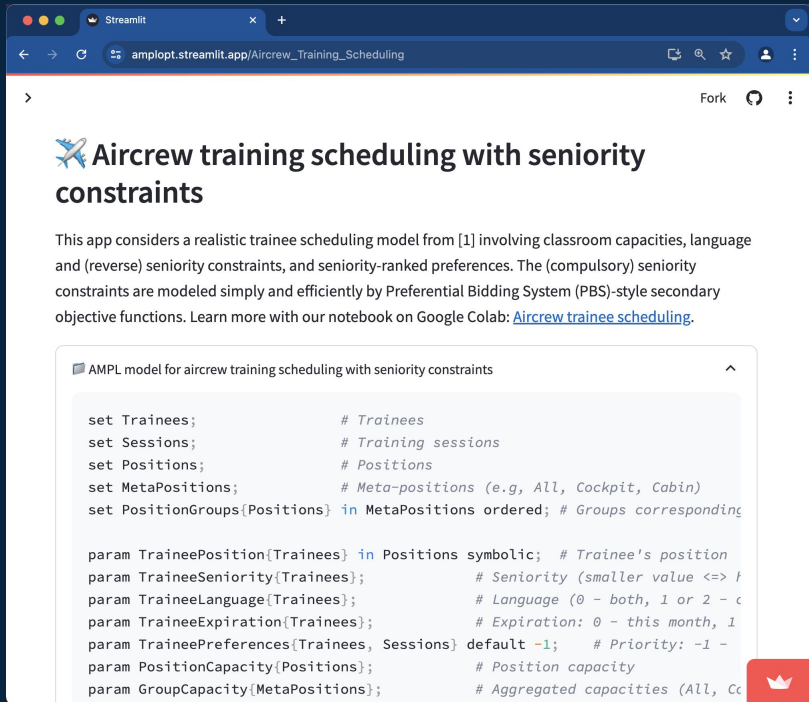
Dr. Oskar Schneider · 1st
AI in Production, Logistics & SCM. Want to learn more about the technology?...
1mo · 🌐
[Case Study] How Fast-Fashion Giant Zara Increased Revenue by \$200'000'000+ with data-driven Decision Making ...more
Every single shipment of Zara's Inventory is determined by Operations Research algorithms
AI in Production, Logistics & SCM
👍👍👍 You and 195 others
31 comments · 12 reposts

LOGISTICS

OPTIMIZATION KPI



Aircrew Training Scheduling (<https://ampl.com/streamlit>)



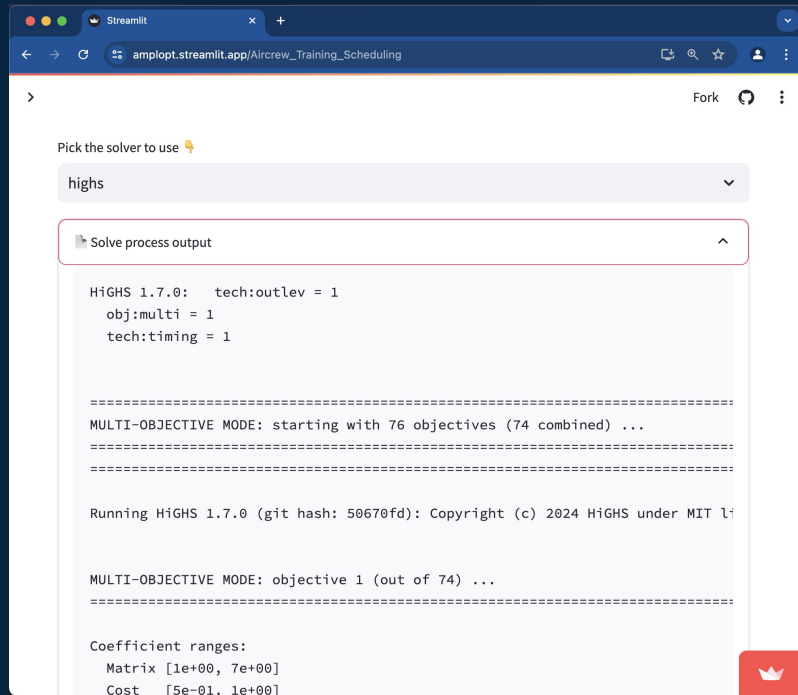
Aircrew training scheduling with seniority constraints

This app considers a realistic trainee scheduling model from [1] involving classroom capacities, language and (reverse) seniority constraints, and seniority-ranked preferences. The (compulsory) seniority constraints are modeled simply and efficiently by Preferential Bidding System (PBS)-style secondary objective functions. Learn more with our notebook on Google Colab: [Aircrew trainee scheduling](#).

AMPL model for aircrew training scheduling with seniority constraints

```
set Trainees;           # Trainees
set Sessions;           # Training sessions
set Positions;          # Positions
set MetaPositions;      # Meta-positions (e.g., All, Cockpit, Cabin)
set PositionGroups[Positions] in MetaPositions ordered; # Groups corresponding

param TraineePosition[Trainees] in Positions symbolic; # Trainee's position
param TraineeSeniority[Trainees];           # Seniority (smaller value <=> higher priority)
param TraineeLanguage[Trainees];           # Language (0 - both, 1 or 2 - specific)
param TraineeExpiration[Trainees];         # Expiration: 0 - this month, 1 - next month
param TraineePreferences[Trainees, Sessions] default -1; # Priority: -1 - low, 0 - medium, 1 - high
param PositionCapacity[Positions];         # Position capacity
param GroupCapacity[MetaPositions];        # Aggregated capacities (All, Cockpit, Cabin)
```



Pick the solver to use

highs

Solve process output

```
HIGHS 1.7.0: tech:outlev = 1
obj:multi = 1
tech:timing = 1

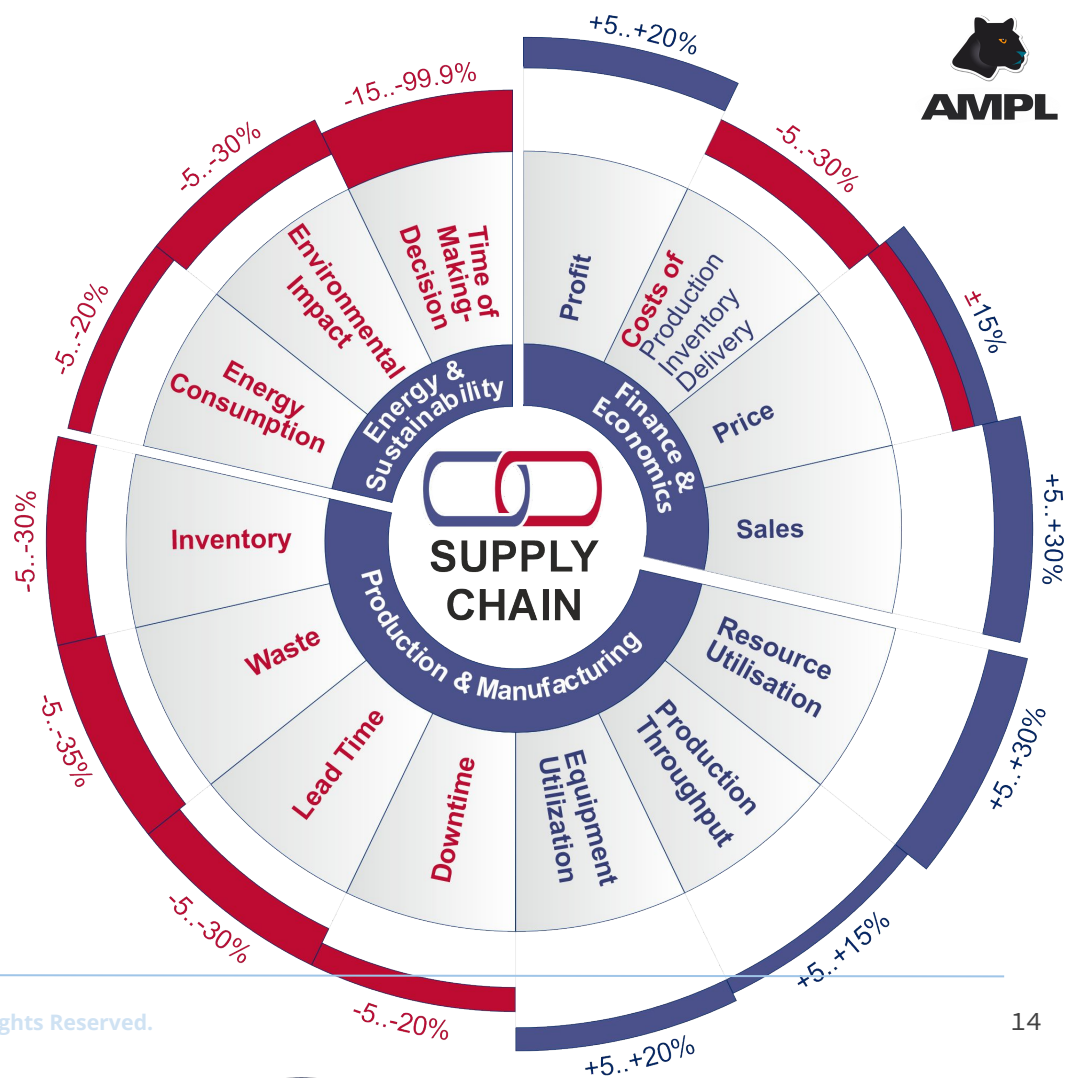
=====
MULTI-OBJECTIVE MODE: starting with 76 objectives (74 combined) ...
=====

Running HIGHS 1.7.0 (git hash: 50670fd): Copyright (c) 2024 HIGHS under MIT license

MULTI-OBJECTIVE MODE: objective 1 (out of 74) ...
=====

Coefficient ranges:
  Matrix [1e+00, 7e+00]
   Cost  [5e-01, 1e+00]
```

SUPPLY CHAIN OPTIMIZATION KPI



Supply Chain App (<https://ampl.com/streamlit>)

The screenshot shows a Google Docs interface with the following content:

Material Balance

$$\text{StartingInventory} + \text{Production Qty} - \text{Demand} = \text{EndingInventory}$$

Class 3 on September 12th: Capacity + Production Hours + transfers + targets + storage constraints + restrict tables

Production and Capacity

$$\text{Production Hrs} * \text{Rate} = \text{Production Qty by Product, Location, Period}$$
$$\text{Production Hrs (by resource)} \leq \text{Available Capacity by Location, Period}$$

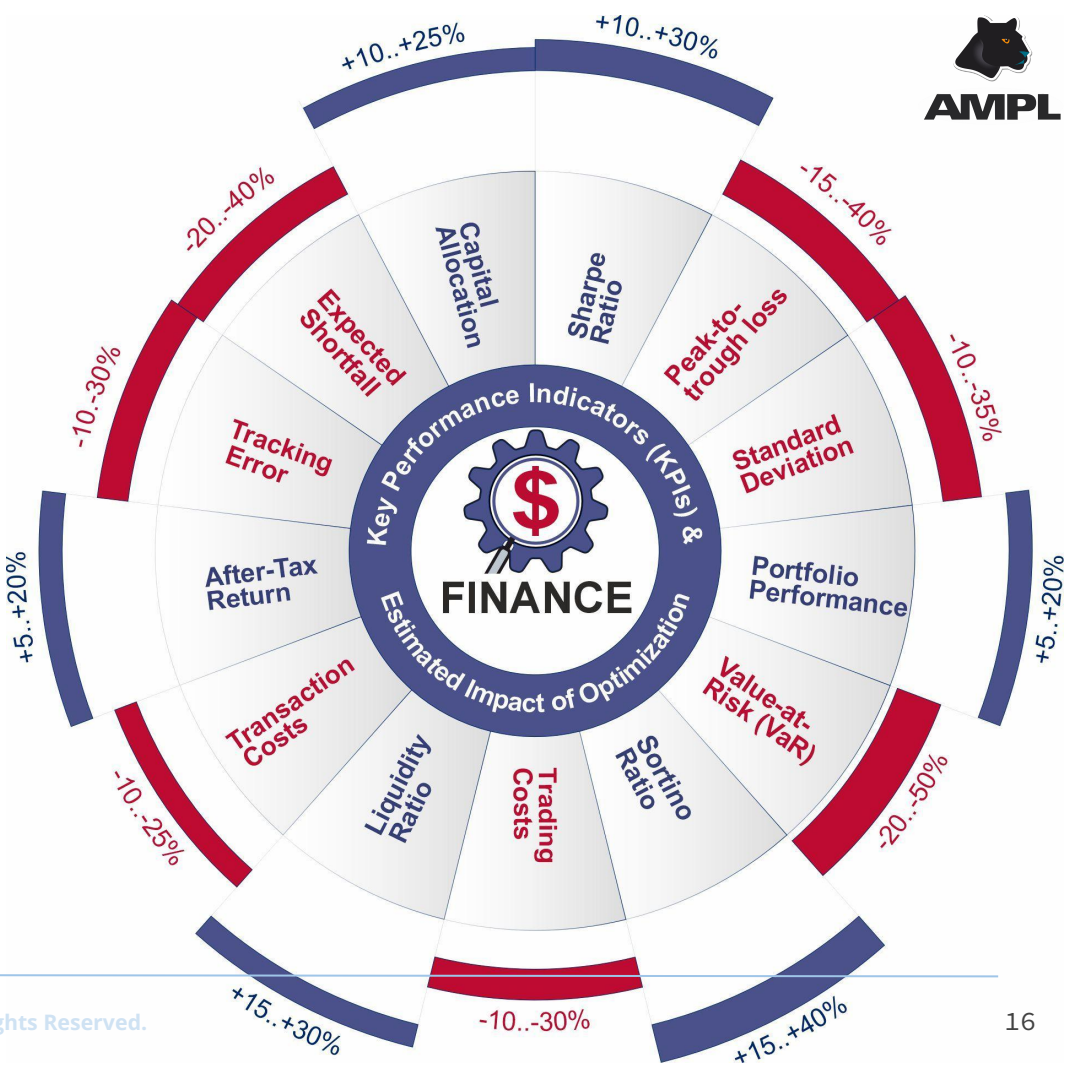
We need a report here with Resource Utilization in which we see the total capacity, used capacity, available capacity (it will be per location as I am modelling resource = location for simplicity)

Transfers

Transfers are added in the Material Balance equation; I want to show here the restrict tables “functionality” for the Transfer Lanes

FINANCE

OPTIMIZATION KPI



finance — AMPL Colaboratory

ampl.com/colab/tags/finance.html

AMPL Colaboratory

Getting Started Modules Highlights Lectures Authors Tags Notebooks

engineering (2 notebooks)

example (7 notebooks)

excel (2 notebooks)

exponential-cone (1 notebook)

facility-location (9 notebooks)

finance (8 notebooks)

formulation-comparison (1 notebook)

gcp (2 notebooks)

gdp (1 notebook)

gecode (1 notebook)

geometry (1 notebook)

geothermal-energy (1 notebook)

gilmore-gomory (1 notebook)

global-optimization (2 notebooks)

google-sheets (2 notebooks)

graphs (4 notebooks)

gui (2 notebooks)

gurobi (5 notebooks)

heuristics (2 notebooks)

highlights (11 notebooks)

Optimized Portfolio Optimization using EIA Data in Python with AMPL

[Notebooks](#) > [Optimized Portfolio Optimization using EIA Data in Python with AMPL](#)

Description: Portfolio Optimization across Crude Oil, Gold, Natural Gas, Silver, and the S&P 500.

Tags: [finance](#), [portfolio-optimization](#), [mean-variance](#)

Author: [Mukeshwaran Baskaran \(4 notebooks\)](#) <mukesh96official@gmail.com>

Pairs Trading Strategy Optimization in Python with AMPL

[Notebooks](#) > [Pairs Trading Strategy Optimization in Python with AMPL](#)

Description: Optimize pairs trading strategy by optimizing entry and exit thresholds for each pair based on training data. This approach uses interpolation to find optimal parameters within the range tested.

Tags: [finance](#), [pairs-trading](#)

Author: [Mukeshwaran Baskaran \(4 notebooks\)](#) <mukesh96official@gmail.com>

[↑ Back to top](#)

Portfolio Optimization with Multiple Risk

On this page

Book Example: Economic equilibria

Efficient Frontier with Google Sheets

Enhanced Sector ETF Portfolio Optimization with Multiple Strategies in Python with AMPL

Financial Portfolio Optimization with amply

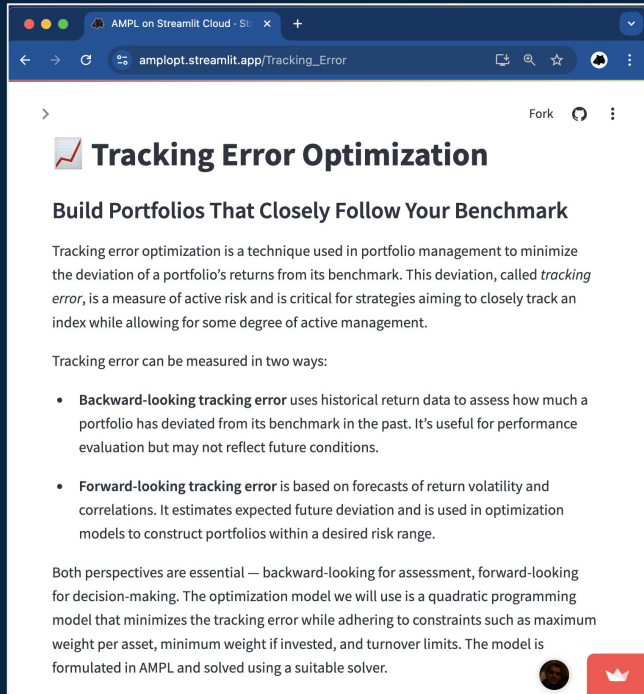
Optimization Methods in Finance: Chapter 3

Optimized Portfolio Optimization using EIA Data in Python with AMPL

Pairs Trading Strategy Optimization in Python with AMPL

Portfolio Optimization with Multiple Risk Strategies in Python with AMPL

Tracking Error Minimization (<https://ampl.com/streamlit>)



Tracking Error Optimization

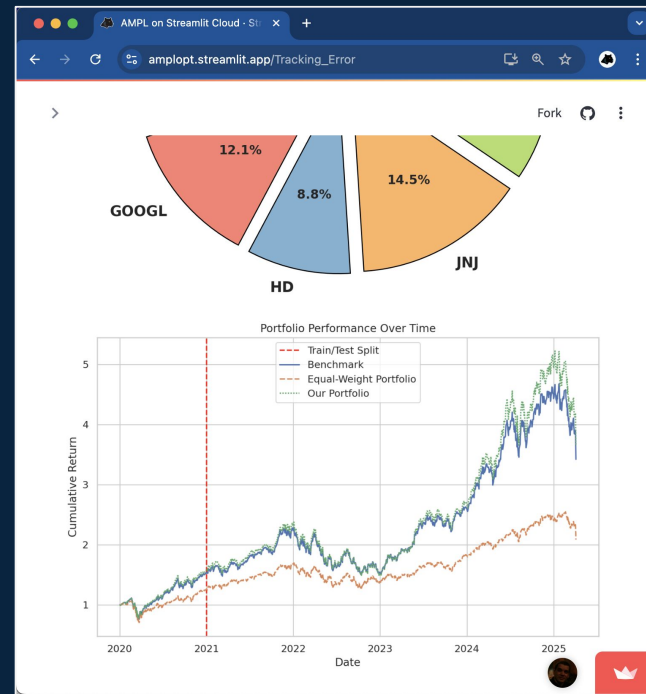
Build Portfolios That Closely Follow Your Benchmark

Tracking error optimization is a technique used in portfolio management to minimize the deviation of a portfolio's returns from its benchmark. This deviation, called *tracking error*, is a measure of active risk and is critical for strategies aiming to closely track an index while allowing for some degree of active management.

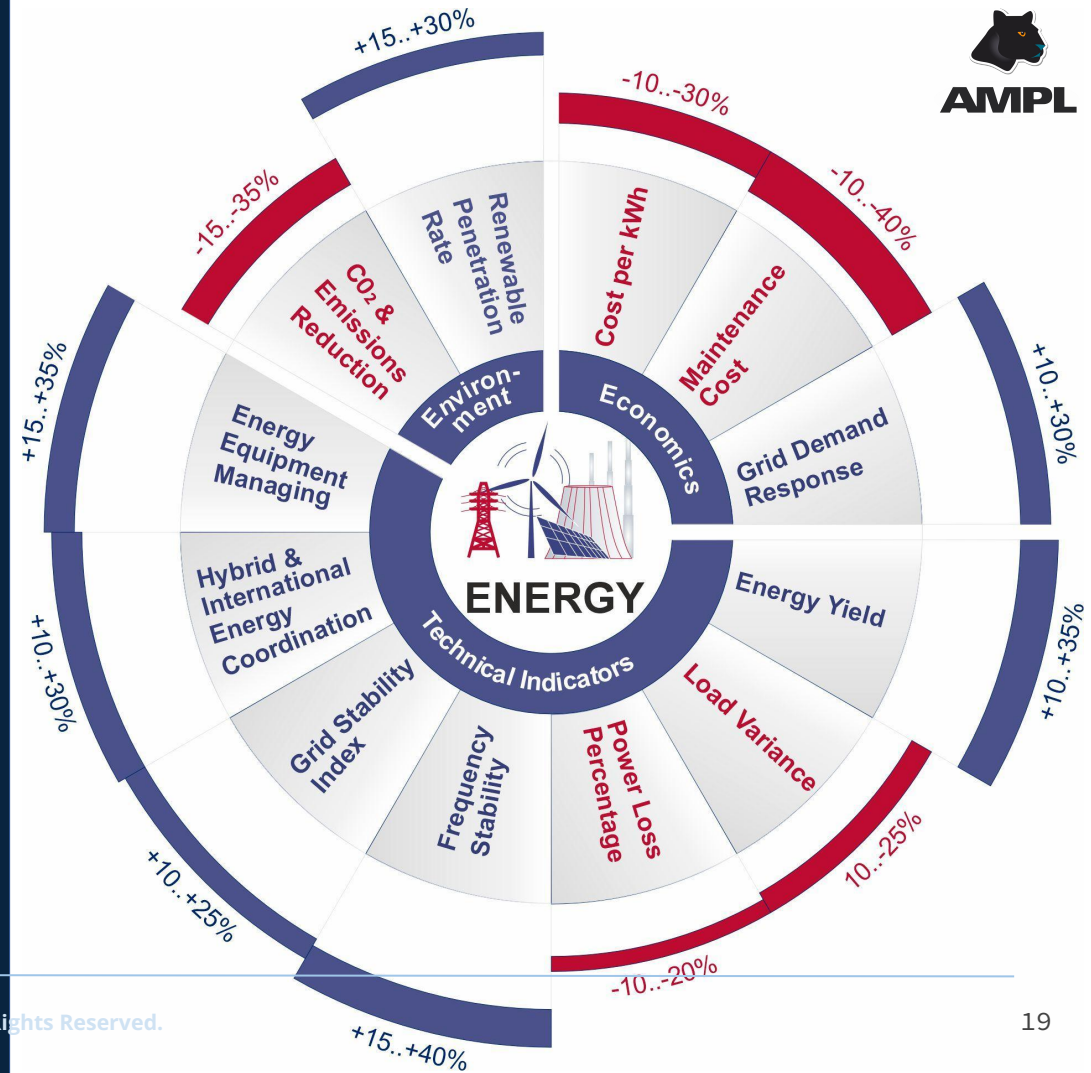
Tracking error can be measured in two ways:

- **Backward-looking tracking error** uses historical return data to assess how much a portfolio has deviated from its benchmark in the past. It's useful for performance evaluation but may not reflect future conditions.
- **Forward-looking tracking error** is based on forecasts of return volatility and correlations. It estimates expected future deviation and is used in optimization models to construct portfolios within a desired risk range.

Both perspectives are essential — backward-looking for assessment, forward-looking for decision-making. The optimization model we will use is a quadratic programming model that minimizes the tracking error while adhering to constraints such as maximum weight per asset, minimum weight if invested, and turnover limits. The model is formulated in AMPL and solved using a suitable solver.



ENERGY OPTIMIZATION KPI



AMPL Industry Use Cases: E

ampl.com/solutions/customers-and-case-studies/case-study-optimizing-power-grid-management-with-ampl-a-case-study-of-hitachi-gridview/

Documentation Academia Consulting Blog Sign in →

AMPL Products Solutions Resources Company Pricing

Start free now Contact sales

← Back to customers and case studies



Optimizing Power Grid Management with AMPL: A Case Study of Hitachi's GridView

CASE STUDY

30+ Companies – 100's Customer-Side users

The integration of AMPL has been a resounding success. GridView, empowered by AMPL's optimization capabilities, is now utilized by over 30 customer companies, serving hundreds of individual users across the power grid management sector. This widespread adoption is a testament to the effectiveness and user-friendliness of the solution.

Implementation and Result of Optimization

The resulting model employs a Mixed-Integer Linear Solver (MILP) to handle a combination of continuous and discrete variables. Despite dealing with millions of variables, including tens of thousands of integer variables, the model achieves optimal solutions within a commendable timeframe of 10 minutes.



Dr. Oskar Schneider · 1st
AI in Production, Logistics & SCM. Want to learn more about the technology?...
5mo · 6

[Case Study] How Hitachi Improves Power Grid Efficiency with Operations Research ...more



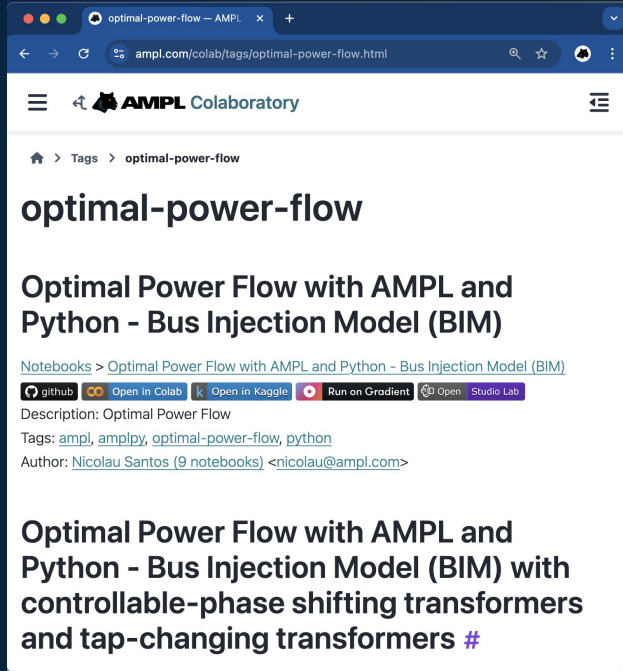
Operations Research Powers Reliable and Cost-Effective Energy Delivery

AI in Production, Logistics & SCM

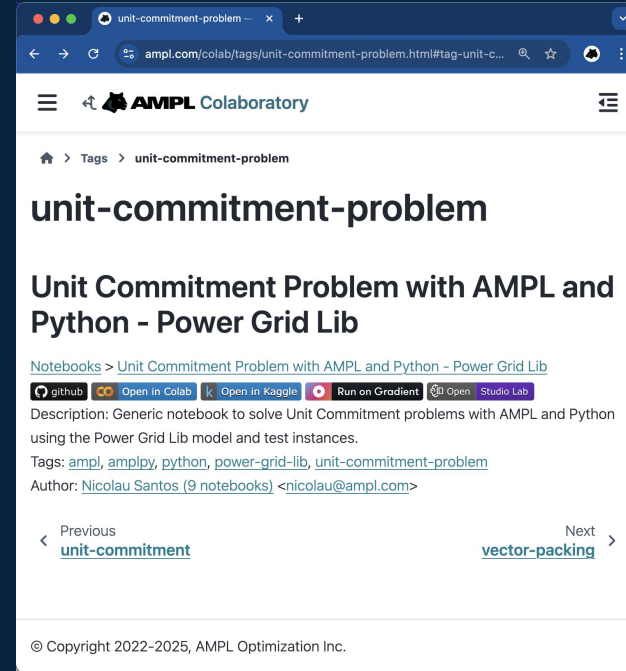
114

6 comments · 8 reposts

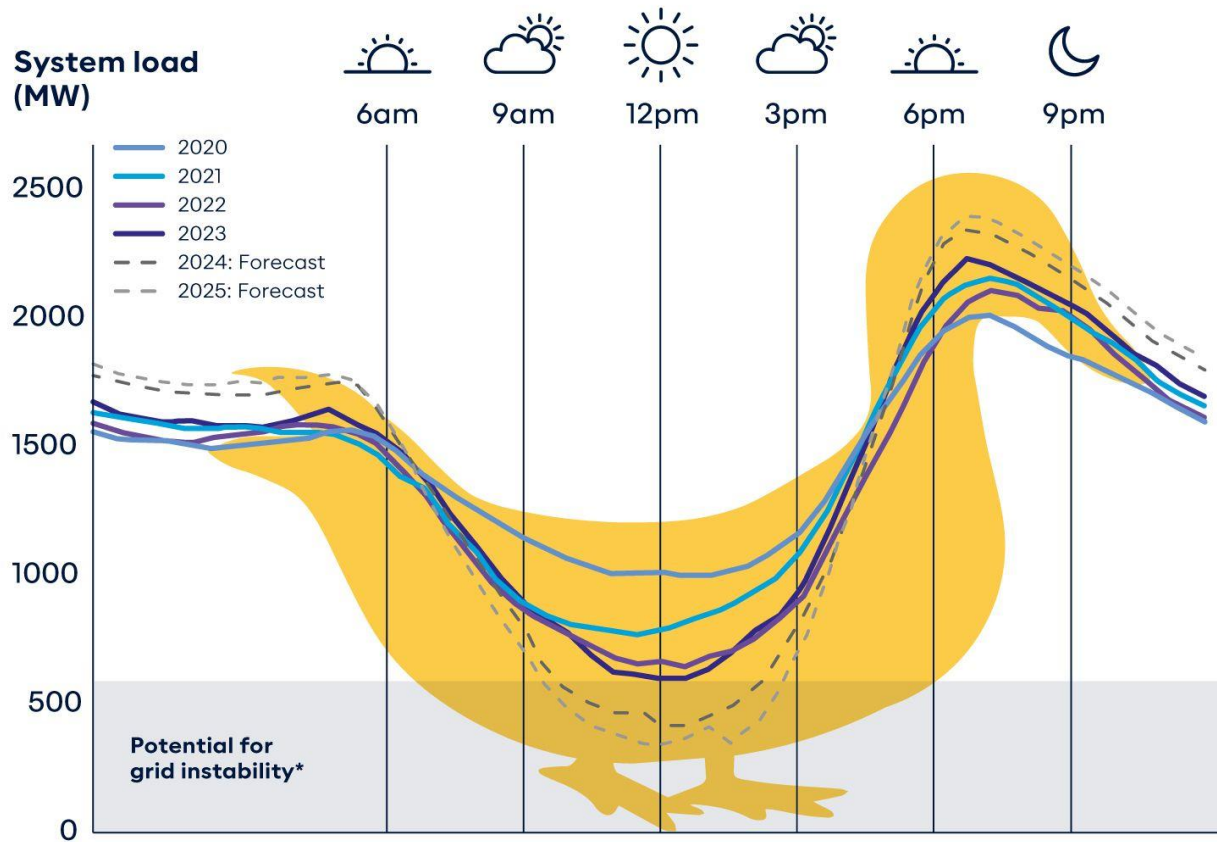
Power Examples on Colab (<https://ampl.com/colab>)



The screenshot shows the AMPL Colaboratory interface for the 'optimal-power-flow' tag. The browser address bar shows 'ampl.com/colab/tags/optimal-power-flow.html'. The page title is 'optimal-power-flow'. Below the title is the subtitle 'Optimal Power Flow with AMPL and Python - Bus Injection Model (BIM)'. There are links to 'Notebooks > Optimal Power Flow with AMPL and Python - Bus Injection Model (BIM)'. Below this are buttons for 'github', 'Open in Colab', 'Open in Kaggle', 'Run on Gradient', 'Open', and 'Studio Lab'. The description is 'Optimal Power Flow'. The tags are 'ampl', 'amplpy', 'optimal-power-flow', and 'python'. The author is 'Nicolau Santos (9 notebooks) <nicolau@ampl.com>'. At the bottom, there is a link to 'Optimal Power Flow with AMPL and Python - Bus Injection Model (BIM) with controllable-phase shifting transformers and tap-changing transformers #'.



The screenshot shows the AMPL Colaboratory interface for the 'unit-commitment-problem' tag. The browser address bar shows 'ampl.com/colab/tags/unit-commitment-problem.html#tag-unit-c...'. The page title is 'unit-commitment-problem'. Below the title is the subtitle 'Unit Commitment Problem with AMPL and Python - Power Grid Lib'. There are links to 'Notebooks > Unit Commitment Problem with AMPL and Python - Power Grid Lib'. Below this are buttons for 'github', 'Open in Colab', 'Open in Kaggle', 'Run on Gradient', 'Open', and 'Studio Lab'. The description is 'Generic notebook to solve Unit Commitment problems with AMPL and Python using the Power Grid Lib model and test instances'. The tags are 'ampl', 'amplpy', 'python', 'power-grid-lib', and 'unit-commitment-problem'. The author is 'Nicolau Santos (9 notebooks) <nicolau@ampl.com>'. At the bottom, there are links for 'Previous unit-commitment' and 'Next vector-packing'. The footer says '© Copyright 2022-2025, AMPL Optimization Inc.'.



Select your filters

Reset filters

Trading Modality

Guarantees of Origin

Market Segment

Auction Name

Delivery Date

05 Apr. 2025

View

Captions

☒ Served PX
☐ Coming soon
☐ EPEX markets

We would like to inform you that we expect a display issue on our website, which may impact the Market Results display for all instruments on the Long Clock Change day. Therefore, we highly recommend referring to the Market Results as published on the Market Data sftp server.

Filters

DAY 00 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24



Auction Status

Last Update: 06 April 2025 (02:48:26 CET/CEST)

MODALITY	STATUS
Intraday 30-call-GB	●
Day-Ahead CH	●
Intraday CH IDA1	●
Intraday CH IDA2	●
Day-Ahead SDAC	●
Intraday IDA1	●
Intraday IDA2	●
Intraday IDA3	●
Day-Ahead GB	●
Intraday GB IDA1	●
Intraday GB IDA2	●

See complete Market status

Operational Messages

Last Update: 06 April 2025 (02:48:26 CET/CEST)

[Privacy settings](#)

Locational Marginal Pricing (<https://ampl.com/streamlit>)

AMPL on Streamlit Cloud - Streamlit

amplpt.streamlit.app/Locational_Marginal_Pricing

⚡ Locational Marginal Pricing

What is Location Marginal Pricing?

Location Marginal Pricing (LMP) is a method used in electricity markets to determine the price of electricity at different locations, or nodes, in the power grid. LMP reflects the marginal cost of supplying the next increment of electricity demand at a specific location, accounting for generation costs, demand, and the physical limitations of the transmission system.

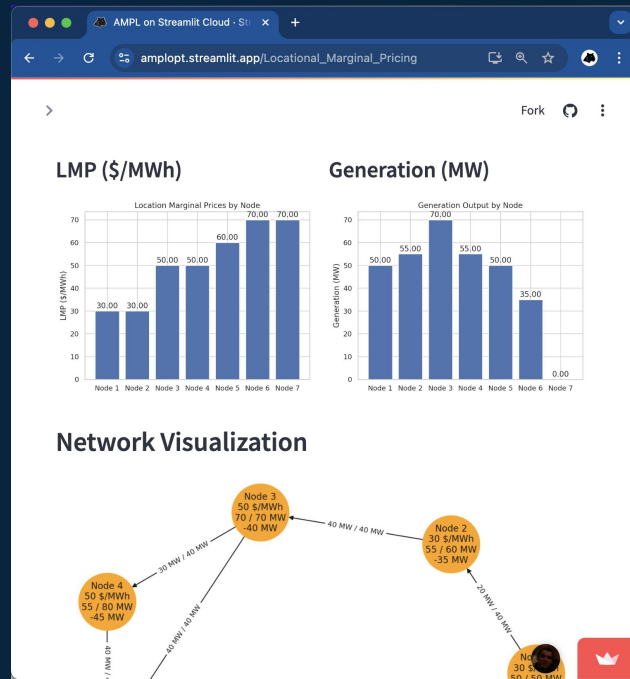
This pricing mechanism ensures that prices signal both energy value and congestion, promoting efficient use of generation and transmission resources.

Learn more on [Optimization Models in Electricity Markets](#) by Anthony Papavasiliou. It is a textbook published by Cambridge University Press that treats the analysis of optimization models that are routinely used in electricity market operations.

Simplified Electricity Market Simulation

```
# Nodes and transmission lines (as node pairs)
set NODES;
set LINES within {NODES, NODES};

# Parameters: generation cost, capacity, demand, and line limits
```



Solving Energy Models with GPUs (<https://ampl.com/cuPDLP>)



AMPL + NVIDIA



CO + HiGS

**GPU-accelerated optimization
available on Google Colab**

ampl.com/cuPDLP



**Breaking Barriers in Optimization:
AMPL's Early Results with NVIDIA cuOpt**

The world of optimization is changing—fast. See the results from early benchmarking of NVIDIA cuOpt with AMPL and what this means for the future of optimization.

ampl.com/start-free-now



ampl.com/blog

Training & Consulting

Accelerate faster with AMPL's Technical Experts

Advance your business confidently with expertise and best practices from the AMPL technical development team. Our services offer the highest level of assurance and proactive investment in the technical health of your optimization projects.

[Talk to an expert →](#)

Model Development & Solver Consulting

We build and refine optimization models for better speed, accuracy, and scalability. Our solver tuning and benchmarking ensure the best solver

Training & Educational Services

We train teams and organizations on best practices, solver optimization, and model development. Universities and researchers get

Industry-specific solutions from expert AMPL consultants

Whether you manually craft or use AI to build your models, our brief consulting service provides comprehensive model accuracy checks,

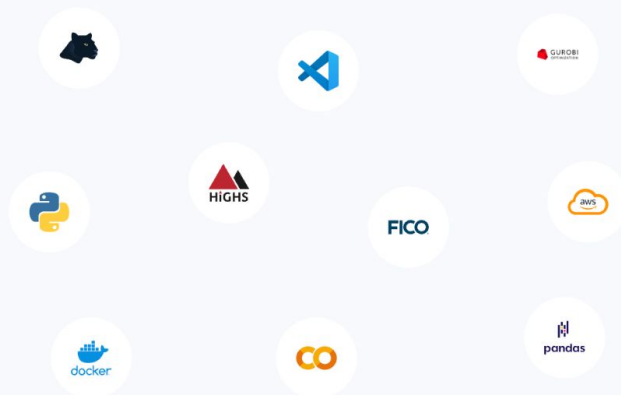
Industry level experience for Academia too

AMPL for Academia

Free Academic Licenses

Industry-leading software for tomorrow's optimization specialists

Empowering the academic community with the same professional modeling system used by global companies. AMPL integrates with Python, VS Code, and modern data stacks – making it easy to go from classroom to production without compromises.



Academic Community License

AMPL for Academics (A4A)

AMPL for Academics (A4A) provides students, faculty, and researchers full AMPL functionality with academic access to select commercial solvers – ideal for coursework, thesis projects,

Academic Teaching License

AMPL for Courses (A4C)

AMPL for Courses (A4C) supports educators in course-based teaching. Easily distributed to entire classes through shareable links and activation codes enabling collaborative use for

Young Professionals License

AMPL Career Starter (ACS)

AMPL Career Starter (ACS) enables recent graduates to take their optimization skills from the classroom to professional practice, bridging the gap between academic study and

AMPL for Academics

Includes:

- Completely free
- No size restrictions
- Full Ampl + All Open source solvers + select Commercial solvers (Gurobi, CPLEX, Mosek, COPT, Xpress (FICO))

Requires:

- Academic email for access (sign-up at portal.ampl.com)

AMPL for Courses

Includes:

- Full AMPL + **all** solvers
- Dynamic cloud licensing by default
 - 1 *uuid* shareable with your course**
- Independent from installation
- Straightforward Google Colab / Python integration

AMPL Career Starter

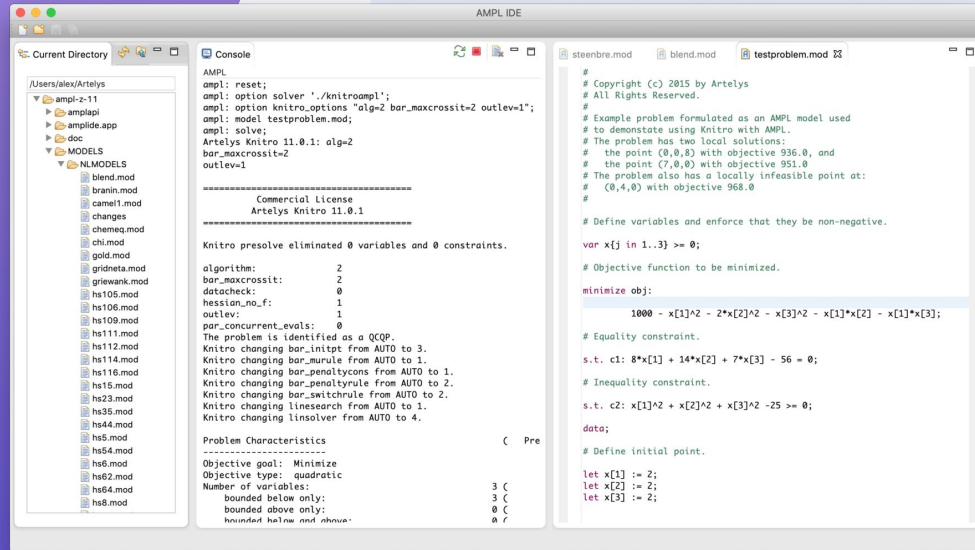
AMPLify your career!

Includes:

- Accessible within 24 months post-graduation for 12-month period
- Full AMPL with open source + commercial solvers
- Commercial/production use for your place of business

Writing optimization in 2025

- Directly from Google Colab
- From wherever you write Python
- Amplide? (**Nope**)



Dark environment IDE ??

On the other hand, I have my AMPL folder in the PATH (environment variables), but I do not know how to run my AMPL files in the VS code terminal.



Hi,

I am a registered user.

May I suggest the inclusion of a color themes option in the IDE?

Including dark options.

Sorry for the inconvenience. I hope you can give me some tips to work on this. I like working in dark environments.

Hello all,

I am looking for a free AMPL editor with dark mode option and syntax highlighting to use in Windows 10.

On other hand, I am looking for a dark mode editor to write my AMPL models, maybe do you know anything about that?

Regards

Visual Studio Code Extension

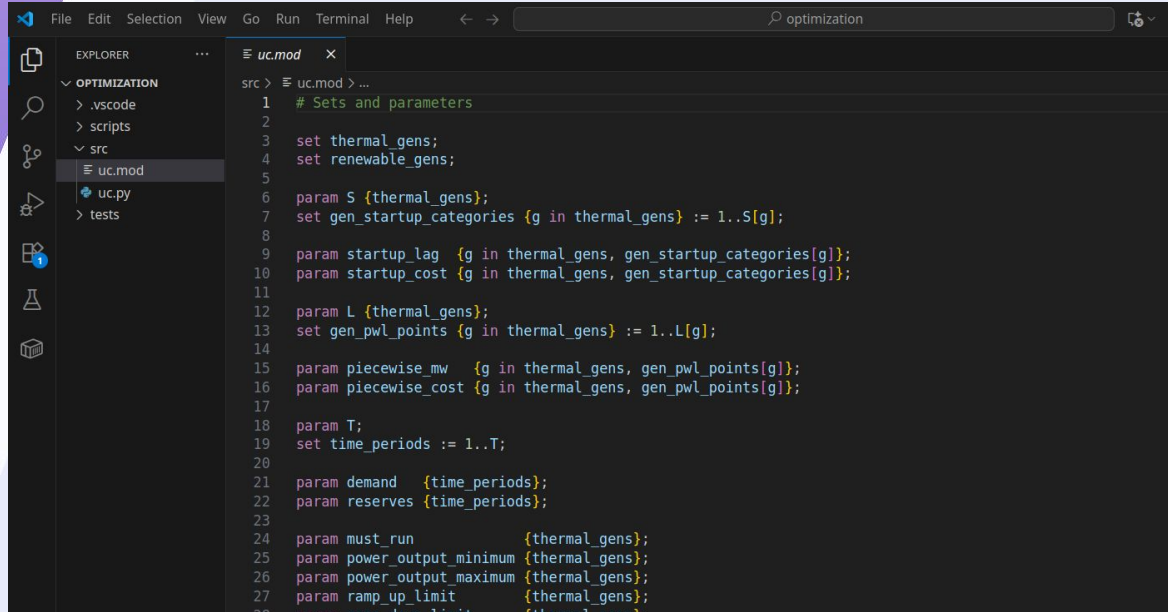
- Develop Python and Ampl code through Visual Studio Code
- Check out the **new official plugin!** [VS Code Marketplace](#)



We recommend using VSCode with an AMPL plugin. Download Here 📌

 [Download Visual Studio Code \(our recommend IDE\)](#)

Visual Studio Code Extension

A screenshot of the Visual Studio Code editor interface. The Explorer sidebar on the left shows a project structure with folders for ".vscode", "scripts", "src", and "tests". The "src" folder is expanded, showing a file named "uc.mod". The main editor window displays the contents of "uc.mod", which is an AMPL model file. The code is written in a dark theme with syntax highlighting. The code includes comments and various AMPL commands such as "set", "param", and "set" for defining sets, parameters, and time periods. The code is as follows:

```
src > uc.mod > ...
1  # Sets and parameters
2
3  set thermal_gens;
4  set renewable_gens;
5
6  param S {thermal_gens};
7  set gen_startup_categories {g in thermal_gens} := 1..S[g];
8
9  param startup_lag {g in thermal_gens, gen_startup_categories[g]};
10 param startup_cost {g in thermal_gens, gen_startup_categories[g]};
11
12 param L {thermal_gens};
13 set gen_pwl_points {g in thermal_gens} := 1..L[g];
14
15 param piecewise_mw {g in thermal_gens, gen_pwl_points[g]};
16 param piecewise_cost {g in thermal_gens, gen_pwl_points[g]};
17
18 param T;
19 set time_periods := 1..T;
20
21 param demand {time_periods};
22 param reserves {time_periods};
23
24 param must_run {thermal_gens};
25 param power_output_minimum {thermal_gens};
26 param power_output_maximum {thermal_gens};
27 param ramp_up_limit {thermal_gens};
28 param ramp_down_limit {thermal_gens};
```

Visual Studio Code Extension

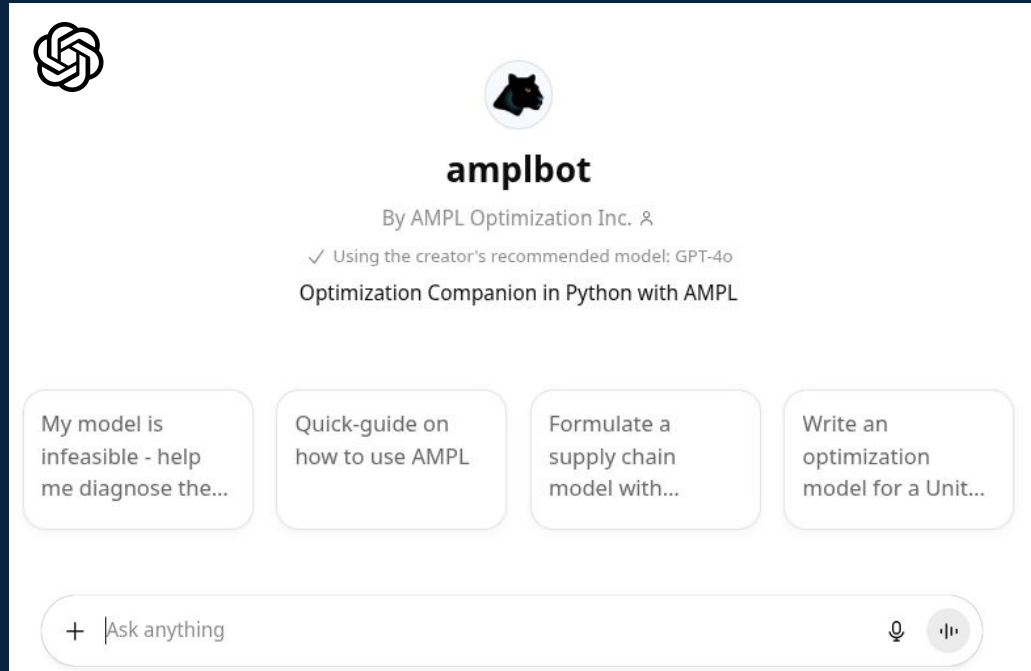


- Syntax highlighting
- Run ampl files
- Display entities and expressions
- Keyboard shortcuts:
 - Run, reset, solve, display expression...
- Search entity declaration

Try the amplbot!

Available at chatgpt.com

- High Level modeling
- Troubleshooting
- Explainability
- Substantial improve over baseline LLMs



AMPL + Python = 

Hands-on: AMPL in Python

Following Workshop resources (check your email, or scan QR code)



- Step 1. Install Amplpy via pip
- Step 2. Activate your license
- Step 3. Run an AMPL model

Even easier, try ampl.com/workshop



Part II

Complex Problems Can Be Made Simple!

Outline Part II

→ Why enhanced modeling?

- User vs. solver perspective

→ Enhanced expression support

- (Automatic) reformulation examples & control options
- MINLP support
- Multi-objective modeling

→ Tools

- Automatic solution checker
- Reformulation Explorer
- Solvers

ChatGPT: “how to formulate a disjunction in MIP?”

1. General Form of a Disjunction

A **disjunction** expresses that **at least one** of several constraint sets must hold:

$$(A_1x \leq b_1) \vee (A_2x \leq b_2)$$

2. Standard MIP Reformulation Using Binary Variables

To encode this in a linear MIP, introduce a **binary variable** $y \in \{0,1\}$.

Then, we can linearize as:

$$\begin{aligned} A_1x &\leq b_1 + M_1y \\ A_2x &\leq b_2 + M_2(1-y) \end{aligned}$$

Here:

- M_1, M_2 are **big-M constants** (large enough to “turn off” the constraints when not active).

ChatGPT: “how to formulate a disjunction in MIP?”

..... (continued)

7. Example in AMPL (Using Logical Constraints via MP Library)

AMPL MP allows disjunctions directly:

```
subject to either_or:
```

```
(x1 <= 5) or (x2 <= 3);
```

→ As a modeler, which would be your preference?

Enhanced modeling: solver perspective

Disjunction modeling

- Most commercial MIP solvers: *indicator constraints*
- Except Mosek: *disjunctive normal form*
- Constraint Programming solvers: arbitrary logical formulas

Solvers claim that their specialized input representations are best for them!

AMPL MP Library: new solver interfaces

Motivation

- Describe optimization models to computers more like you think about them
 - Development speed, transparency, maintainability
 - Better/more general information for the solver => performance ?

Strategy

- Extend modeling tools to more natural objective and constraint expressions
- Build solver interfaces to convert natural expressions to forms that solvers require

Supported Expressions

AMPL operators and functions

- Conditional: **if-then-else**; **==>**, **<==**, **<==>**
- Logical: **or**, **and**, **not**; **exists**, **forall**
- Piecewise linear: **abs**; **min**, **max**; **<<breakpoints; slopes>>**
- Counting: **count**; **atmost**, **atleast**, **exactly**; **numberof**
- Comparison: **>**, **<**, **!=**; **alldiff**
- Complementarity: **complements**
- Nonlinear: *****, **/**, **^**; **exp**, **log**; **sin**, **cos**, **tan**; **sinh**, **cosh**, **tanh**
- Set membership: **in**

Recognizable expressions

- High-order polynomials
- Second-order cones
- Exponential cones (MOSEK driver!)

A basic case: indicator (into inequality)

```
var b binary;  
s.t. Indicator: b==1 ==> 5*x[1] + 2*x[2] <= 0;
```

- **Natively supported by many solvers**
- **Important: tight bounds on the compared expression, e.g.:**

```
var x {1..2} >=0 <=15;
```

- **Linearized when not supported, or when desired (option `acc:indle=0`):**

```
s.t. BigM: 5*x[1] + 2*x[2] <= 105*(1-b);
```

Disjunction: $\vee$, exists

subject to Disjunction:

$(\text{level_A} \leq 5) \vee (\text{level_B} \geq 10);$

Reformulation for a typical MIP solver using indicators:

```
var {1..3} aux_result: binary;  
s.t. Term1: aux_result[1] ==> level_A <= 5;  
s.t. Term2: aux_result[2] ==> level_B >= 10;  
s.t. Disj__:  
    aux_result[3] ==> (aux_result[1] || aux_result[2]);  
s.t. FixResult: aux_result[3] <==> True;
```

Indicators and OR are linearized if necessary.

$\implies$ vs $\iff$ vs $\leq$: when which?

```
minimize TotalCost:
```

```
    if (level_A <= 5) || (level_B >= 10) then 1000;  # else 0
```

Reformulation:

```
var {1..3} aux_result: binary;
```

```
s.t. Term1: aux_result[1] <= level_A <= 5;
```

```
s.t. Term2: aux_result[2] <= level_B >= 10;
```

```
s.t. Disj__:
```

```
    aux_result[3] <= (aux_result[1] || aux_result[2]);
```

```
minimize TotalCost__: 1000*aux_result[3];
```

The disjunction is in *negative context* which reverses the implications.

An experiment with max()

From S. Brand et al. (2008), *Flexible, rule-based constraint model linearisation*, citing a radiation model for intensity-modulated radiotherapy, core part: min-cardinality decomposition in C1P matrices:

```
subject to Increment_Constraints {i in ROWS, b in BTIMES}:  
    N[b] >= Q[i,1,b]  
        + sum {j in 2..n} max(Q[i,j,b] - Q[i,j-1,b], 0);
```

- The max() expressions are in negative context: sufficient linearization is

```
var Z {i,j,b} >=0;  
s.t. LinearizeNegCtx {i,j,b}:  
    Z[i,j,b] >= Q[i,j,b] - Q[i,j-1,b];  
subject to Increment_Constraints_Lin {i in ROWS, b in BTIMES}:  
    N[b] >= Q[i,1,b]  
        + sum {j in 2..n} Z[i,j,b];
```

An experiment with max()

```
subject to Increment_Constraints{i in ROWS, b in BTIMES}:  
    N[b] >= Q[i,1,b] + sum{j in 2..n} max(Q[i,j,b] - Q[i,j-1,b], 0);
```

- The max() expressions are in negative context: sufficient linearization is

```
var Z {i,j,b} >=0;  
s.t. LinearizeNegCtx {i,j,b}: Z[i,j,b] >= Q[i,j,b] - Q[i,j-1,b];
```

- Authors observe that CPLEX 9.1 performs much better with full linearization which includes positive context:

```
var B {i,j,b,1..2} binary;  
s.t. Disjunction {i,j,b}: B[i,j,b,1] + B[i,j,b,2] >= 1;  
s.t. LinearizePosCtx {i,j,b}:  
    B[i,j,b,1]==1 ==> Z[i,j,b] <= Q[i,j,b] - Q[i,j-1,b]  
    && B[i,j,b,1]==1 ==> Z[i,j,b] <= 0;
```

Replicate the experiment

		Native max()		Linear max()		Linear max(), full	
		N solved	mean t	N solved	mean t	N solved	mean t
Max. intens. 10	CPLEX 22.1.1	-	-	10	1.80	10	2.53
	Gurobi 12.0.3	10	0.81	10	1.02	10	1.06
	Highs 1.11	-	-	10	3.33	10	6.37
Max. intens. 12	CPLEX	-	-	10	18.47	10	26.05
	Gurobi	10	6.21	10	10.27	9	33.50
	Highs	-	-	10	12.18	10	20.69

Apple M1 Pro, 1 thread, 10 instances per category, time limit 300s

Discussion

- Observations contrast those from S. Brand et al. (2008):
 - Gurobi:
 - Native `max()` is best, followed by simple linearization. Full linearization is worst.
 - CPLEX 22.1.1, Highs:
 - Simple (context-aware) linearization is best, as expected.
- Options to control reformulations (dev.ampl.com):
 - `acc:max=4` as expression tree node, if supported
 - `acc:max=2` as general constraint, if supported
 - `acc:max=0` `linearize` (context-aware by default)
 - `cvt:pre:ctx:max=0` full linearization

Corner Case: Strict Comparison

Consider the above reverse implication:

```
s.t. Term1: aux_result[1] <== level_A <= 5;
```

Which is equivalent to the indicator constraint

```
Term1IndStrict: (aux_result[1]==0) ==> (level_A > 5);
```

Assuming `level_A` is real-valued, what is the meaning of `(level_A > 5)`?

- For the modeler...
- For the chip...

Strict comparison tolerance

```
IndStrict: (aux_result[1]==0) ==> (level_A > 5);
```

AMPL MP converts strict comparisons for MIP solvers into non-strict comparisons using option `cmp:mip:eps` (default value `1e-4`):

```
IndNonStrict: (aux_result[1]==0) ==> (level_A >= 5.0001);
```

- A guideline for the value of `cmp:mip:eps` is to set it at least 10x larger as `feastol`, the primal feasibility tolerance (default `1e-6` for most solvers).
- When the comparison has a single context (pos/neg), the strictness can be chosen to avoid tolerance application:

```
minimize TotalCost_NoFinalStrictCmp:
```

```
    if (level_A < 5) || (level_B > 10) then 1000;
```

(Dis)equalities

- Implied equality: again, an indicator

s.t. IndEq: $b=0 \implies 5x[1] + 2x[2] == 7;$

- Reverse implication:

s.t. IndEqRev0: $b=0 \leq 5x[1] + 2x[2] == 7;$

- Equivalent to implied disequality

s.t. IndDiseq: $b=1 \implies 5x[1] + 2x[2] != 7;$

- Reformulation as a disjunction:

$\dots \implies (5x[1] + 2x[2] < 7 \text{ or } 5x[1] + 2x[2] > 7);$

- *Unary encoding* is applied in some cases

Unary encoding

```
var Final_patt_seq {SLOT_SEQ} in PATTERNS_ALL;    # integer vars
s.t. assign_color_SLV {r in ROUND, s in SLOT, k in PATTERNS: ...}:
    Final_patt_seq[slot_r_cum[r] + s]==k
    ==> exists {i in 1..noSLV}
        (Color_SLV[slot_r_cum[r] + s,i]==color_spec1[k]
         and Avail_SLV[slot_r_cum[r] + s,i]);
```

- When an integer variable compares to several values, unary encoding applies:

```
var x in 1..3;
translates into
var unary {1..3} binary;
x == unary[1] + 2*unary[2] + 3*unary[3];
1 == sum {i in 1..3} unary[i];
```

Options `uenc:ratio` and `uenc:negctx` control the choice between reformulations.

Operators count, atleast/atmost/exactly, numberof

Generalize disjunction to count the number of true terms:

```
minimize NumBigDeviations:  
    count {i in ASSETS}  
        ( abs( weight[i]-benchmark[i] ) > 0.005 );  
s.t. LimitBigWeights:  
    atmost 5 {i in ASSETS} ( weight[i] > 0.05 );
```

Finance AMPL Colab notebooks: <https://colab.ampl.com/tags/finance.html>

The Golomb Ruler problem

A *Golomb ruler* is a set of marks at integer positions along a ruler such that no two pairs of marks are the same distance apart.

```
param m default 4;
```

```
param n = m*m;
```

```
set OrdPairs = {i in 1..m-1, j in 2..m: i<j};
```

```
var mark {i in 1..m} in 0..n;
```

```
var differences {(i,j) in OrdPairs} in 1..n;
```

```
s.t. AssignDiffs {(i,j) in OrdPairs}:
```

```
    differences[i,j] == mark[j] - mark[i];
```

```
s.t. FixMark1: mark[1] == 0;
```

```
s.t. MarksOrdered {i in 1..m-1}: mark[i] <= mark[i+1]-1;
```

The Golomb Ruler problem

```
param DefaultAllDiff default 1;          ## Choose the alldiff
s.t. AllDiff {if DefaultAllDiff}:        ## MIP reform. uses UEnc
    alldiff {(i,j) in OrdPairs} differences[i,j];    ## CP: native

s.t. AllDiffNaive {if not DefaultAllDiff}:
    forall {(i,j) in OrdPairs, (k,l) in OrdPairs:
        i<k || (i==k && j<l)}
        differences[i,j] != differences[k,l];

s.t. AntiSymm3: mark[2] - mark[1] <= mark[m] - mark[m-1] - 1;

minimize Largest: mark[m];
```

Golomb Ruler

m (opt)		Native alldiff		alldiff -> Uenc		Naïve alldiff	
		nodes/cp	t	nodes/cp	t	nodes/cp	t
7 (25)	IBM ILOG CP	1693	0.02	96782	2.35	1693	0.02
	Gurobi	-	-	985	2.94	1533	0.47
	Highs	-	-	660	2.57	1282	1.26
9 (44)	IBM ILOG CP	149818	1.82	3613742	269.85	149818	1.81
	Gurobi	-	-	4437	69.81	18463	82.32
	Highs	-	-	34194	154.75	286620	508.96

Apple M1 Pro, 1 thread

Golomb Ruler OLD

		alldiff		alldiff naive	
m (opt)		nodes/cp	t	nodes/cp	t
7 (25)	IBM ILOG CP	1693	0.02	1693	0.02
	Gurobi	985	2.94	1533	0.47
	Highs	660	2.57	1282	1.26
9 (44)	IBM ILOG CP	149818	1.82	149818	1.81
	Gurobi	4437	69.81	18463	82.32
	Highs	34194	154.75	286620	508.96

Apple M1 Pro, 1 thread

Reformulation control essentials (dev.ampl.com)

- Options `acc:max`, `acc:abs`, `cvt:pre:ctx:cos`, etc.
 - Native constraint/expression acceptance and context
- Option `cvt:mip:eps`
 - Real-valued strict comparison tolerance
- Option `cvt:bigM`
 - Default big-M value for unbounded variables, when linearization is necessary or desired. Avoid unbounded variables.
- Options `cvt:compl[:eps]`
 - Complementarity reformulations
- Options `cvt:plapprox:...`
 - Domain and precision of piecewise-linear approximation of nonlinear functions

(MI)NLP support for MP-based solvers

(MI)NLP support for Gurobi, Xpress Global, COPT (NLP only), BaronMP, and SCIP

```
minimize Fchi:  
    x[1]^2 - 12*x[1] + 11 + 10*cos(pi*x[1]/2)  
    + 8*sin(pi*5*x[1]) - exp(-(x[2] - .5)^2/2)/sqrt(5);
```

- Disabled in COPT by default, set
acc:_expr=2 [alg:relax=1]

Multiobjective modeling

- Multiobjective models can be simpler for certain problems
 - Some constraints can be moved to the objectives as penalties
 - Lexicographic optimization allows objective ranking
- Example on AMPL Colab: aircrew trainee scheduling with seniority constraints <https://colab.ampl.com/tags/multi-objective.html>
 - Smaller model and faster solving than the original hard-constrained model

```
maximize ReverseSeniority {e in 1..2, i in I: E[i]==e}:  
  sum {t in V[i]: Pr[i, t]==0}  
    (S[i] - min {j in I} S[j] + 1) * x[i, t]  
  suffix objpriority (2-e)*S_range + 1 + S[i] - min {j in I} S[j];
```

Automatic solution checker

- All MP solvers automatically check every solution
- Tolerance options
 - `sol:chk:feastol`, `sol:chk:feastolrel` (both default `1e-6`)
 - `sol:chk:inttol` (default `1e-5`) integrality
 - Warning on violation, or abort if option `sol:chk:fail` given
- Checks only original (AMPL-side) and final (solver-side) model items by default
 - Option `sol:chk:mode` can add intermediate reformulated expressions

<https://mp.ampl.com/modeling-tools.html#automatic-solution-check>

Check solutions yourself!

E.g., in AMPL:

- Objective value(s):

```
_obj
```

- Algebraic constraints and variable bounds:

```
min{i in 1 .. ncons} con[i].slack      # Should be >= -1e-6  
min{i in 1 .. _nvars} _var[i].slack
```

- Logical constraints:

```
if forall {i in 1.._nlogcons} _logcon[i] then 1
```

- Complementarities:

```
max{i in 1 .. _ncons} abs(_ccon[i])    # Should be <= 1e-6
```

Reformulation Explorer

- Want to know what AMPL MP does to your model?
 - Simplest way: option writeprob=[model.lp](#) or similar
 - Exports solver model in the LP or another format
 - To see intermediate reformulation steps, use <https://mp.ampl.com/modeling-tools.html#reformulation-explorer>.
- ▼ NL Objectives (1) {1}
- ▼ minimize TotalCost: if x < 5 || y > 10 then 1000; {10}
 - ▶ minimize TotalCost____: 1000*TotalCost__6____; {0}
 - ▶ var TotalCost__2____ binary; {0}
 - ▼ TotalCost__3____: TotalCost__2____==1 <==> (x____ < 5); {1}
 - ▶ TotalCost__3____: TotalCost__2____==0 ==> (x____ >= 5);
 - ▶ var TotalCost__4____ binary; {0}
 - ▶ TotalCost__5____: TotalCost__4____==1 <==> (y____ > 10); {1}

NL reader and writer libraries

- NL is a low-level model format accepted by AMPL solvers and used by several modeling systems
- NL reader and writer libraries in MP use low-overhead C++ template-based interfaces
- C and Python NL writer wrappers are available for MILP
 - Package nlwpy on PyPi

Which solver to use?

(Originally) Linear solvers

[1]

GPU-accelerated
algorithms, such as
PDLP, are
available.

[2]

Conic
programming:
MOSEK supports
SOCP and
exponential cones,
other solvers only
SOCP.

	LP	MILP	QP	MIQP	convex (MI)QCP	non-convex (MI)QCP	Conic	MINLP	<u>MP</u>
<u>Gurobi</u>	✓	✓	✓	✓	✓	✓	✓	✓	✓
<u>FICO XPRESS</u>	✓	✓	✓	✓	✓	✓	✓	✓	✓
<u>IBM CPLEX</u>	✓	✓	✓	✓	✓	✗	✓	✗	✓
<u>COPT</u>	✓ ^[1]	✓	✓	✓	✓	✗	✓	✗	✓
<u>MOSEK</u>	✓	✓	✓	✓	✓	✗	✓ ^[2]	✗	✓
<u>NVIDIA cuOpt</u>	✓ ^[1]	✓ ^[1]	✗	✗	✗	✗	✗	✗	✓
<u>HiGHS</u>	✓ ^[1]	✓	✓	✗	✗	✗	✗	✗	✓
<u>CBC</u>	✓	✓	✓	✓	✗	✗	✗	✗	✓
<u>SCIP</u>	✓	✓	✓	✓	✓	✓	✓	✓	✓
<u>GCG</u>	✓	✓	✓	✓	✓	✓	✓	✓	✓

Nonlinear solvers

[3]

Global optimality:
while Knitro and
Bonmin accept
integer variables,
they only guarantee
local optimum for
non-convex models.

[4]

With the MP2NL
meta-driver.

	NLP	MINLP	Global [3]	MP
Knitro	✓	✓	✗	✓ [4]
BARON	✓	✓	✓	✓
LINDO Global Solver	✓	✓	✓	✓ [4]
Octeract	✓	✓	✓	✓ [4]
RAPOSa	✓	✓	✓	✓ [4]
LGO	✓	✗	✓	✓ [4]
CONOPT	✓	✗	✗	✓ [4]
LOQO	✓	✗	✗	✓ [4]
MINOS	✓	✗	✗	✓ [4]
SNOPT	✓	✗	✗	✓ [4]
IPOPT	✓	✗	✗	✓ [4]
BONMIN	✓	✓	✗	✓ [4]
COUENNE	✓	✓	✓	✓ [4]

AMPL Constraint Programming solvers

- IBM ILOG CP
- Gecode

CP solvers support most combinatorial MP constructs natively.

MP2NL meta-solver

- MP2NL enables MP features to other AMPL solvers that have not been reimplemented with the MP library
- While could be most useful for MINLP solvers (Knitro, Baron, Couenne), MP2NL enables complementarity constraints and (convex) piecewise-linear expressions with NLP solvers, such as Ipopt and Conopt

```
AMPL.solve(solver='mp2nl',  
           ipopt_options='max cpu time 15',  
           mp2nl_options='solver=ipopt cvt:compl=2 cvt:compl:eps=1e-8')
```

<https://mp.ampl.com/modeling-tools.html#meta-driver-mp2nl>

Summary

- AMPL+MP: bridge between modelers and solvers
 - Write models more like you think about them
 - Solvers receive the preferred model form
- “Set and forget”: our approach to modern real-world optimization development
- Check the [Workshop resources](#) for more!

Thank you!

Visit us @ Booth 221

Technology Showcase: *From Classroom to Industry:
Modern Optimization with AMPL*

Tuesday, October 28 | 11:00 AM - 12:15 PM

Building A Level 3 A301

Technical session: *Efficient Data Exchange in Amplpy
via Apache Arrow Integration.* Jurgen Lentz

Tuesday, October 28 | 2:45 PM - 4:00 PM

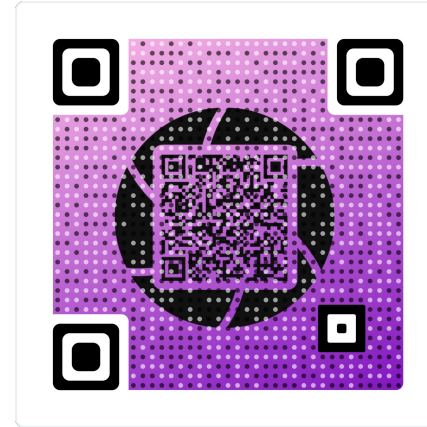
Building B Level 2 B201

Contact us:

academia@ampl.com

AMPL Academic Community:

<https://discuss.ampl.com/>



— — —

Supported Solver Features and Emulated Features

(<https://mp.ampl.com/features-guide.html>)

The screenshot shows the 'Features guide for MP-based' page on the AMPL MP website. The page title is 'Solvers support'. A table summarizes the solver driver support for each solver feature. The table has 10 columns for solvers: Copt, CPLEX, Gurobi, Mosek, Xpress, CBC, GCG, HIGHS, and SCIP. The rows list features: Model export, Solution Export, Report solution time, Feasibility Relaxation, iis, Input and output basis, Kappa, and Multiple solutions. Green checkmarks indicate supported features, while red X's indicate unsupported features. A note states that features marked with a checkmark are supported through emulation.

Solvers support

This table summarizes the solver driver support for each solver feature; some features, denoted by ✓ are supported through emulation (aka they were not available in the original solver but are emulated by MP).

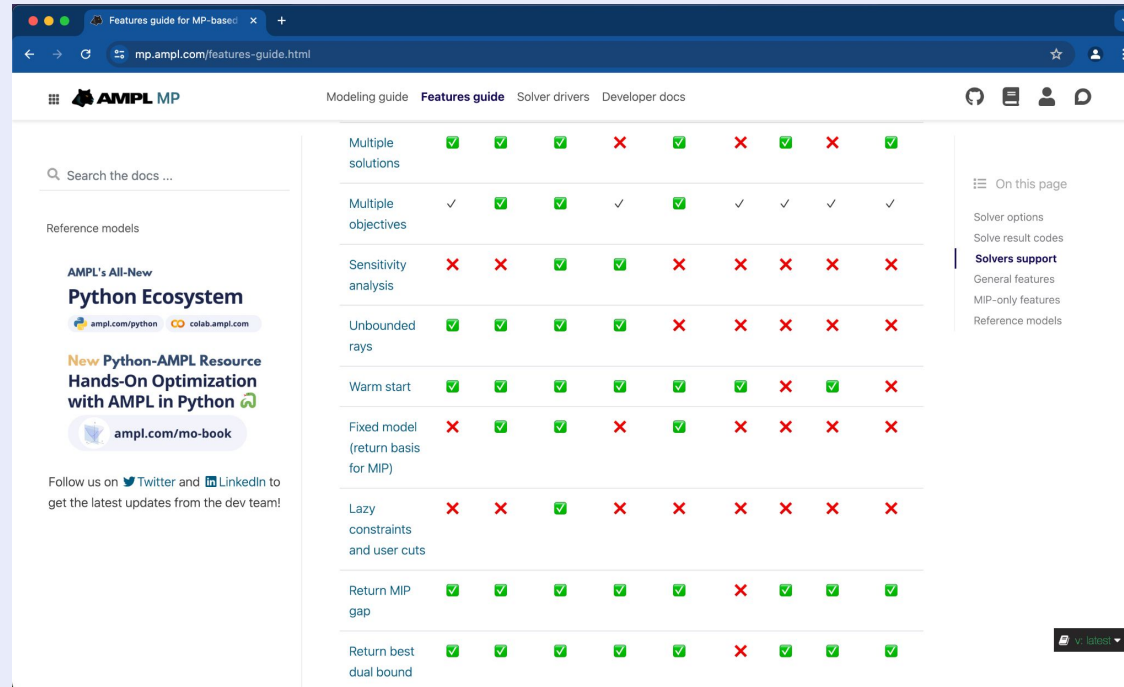
Feature	Copt	CPLEX	Gurobi	Mosek	Xpress	CBC	GCG	HIGHS	SCIP
Model export	✓	✓	✓	✓	✓	✓	✓	✓	✓
Solution Export	✗	✓	✓	✗	✓	✗	✗	✓	✗
Report solution time	✓	✓	✓	✓	✓	✓	✓	✓	✓
Feasibility Relaxation	✓	✓	✓	✗	✗	✗	✗	✗	✗
iis	✓	✓	✓	✗	✓	✗	✗	✗	✗
Input and output basis	✓	✓	✓	✓	✓	✓	✓	✓	✗
Kappa	✗	✓	✓	✗	✗	✗	✗	✗	✗
Multiple solutions	✓	✓	✓	✗	✓	✗	✓	✗	✓

On this page:

- Solver options
- Solve result codes
- Solvers support**
- General features
- MIP-only features
- Reference models

Supported Solver Features and Emulated Features: the list goes on...

(<https://mp.ampl.com/features-guide.html>)



The screenshot shows the 'Features guide' page for AMPL MP. The page has a navigation bar with 'Modeling guide', 'Features guide' (selected), 'Solver drivers', and 'Developer docs'. On the left, there is a search bar and a sidebar with links to 'Reference models', 'AMPL's All-New Python Ecosystem', 'New Python-AMPL Resource Hands-On Optimization with AMPL in Python', and 'Follow us on Twitter and LinkedIn'. The main content area displays a table of features for 10 solvers: CPLEX, Gurobi, Xpress, SCIP, Couenne, Knitro, BARON, Lindero, and AMPL. Features are marked as supported with a green checkmark and as emulated with a red X.

Feature	CPLEX	Gurobi	Xpress	SCIP	Couenne	Knitro	BARON	Lindero	AMPL
Multiple solutions	✓	✓	✓	✗	✓	✗	✓	✗	✓
Multiple objectives	✓	✓	✓	✓	✓	✓	✓	✓	✓
Sensitivity analysis	✗	✗	✓	✓	✗	✗	✗	✗	✗
Unbounded rays	✓	✓	✓	✓	✗	✗	✗	✗	✗
Warm start	✓	✓	✓	✓	✓	✓	✗	✓	✗
Fixed model (return basis for MIP)	✗	✓	✓	✗	✓	✗	✗	✗	✗
Lazy constraints and user cuts	✗	✗	✓	✗	✗	✗	✗	✗	✗
Return MIP gap	✓	✓	✓	✓	✓	✗	✓	✓	✓
Return best dual bound	✓	✓	✓	✓	✓	✗	✓	✓	✓