

# Specifying “Logical” Conditions in AMPL Optimization Models



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AMPL Optimization

[www.ampl.com](http://www.ampl.com) — 773-336-AMPL

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# **New and Forthcoming Developments in the AMPL Modeling Language and System**

Optimization modelers are often stymied by the complications of converting problem logic into algebraic constraints suitable for solvers. The AMPL modeling language thus allows various logical conditions to be described directly. Additionally a new interface to the ILOG CP solver handles logic in a natural way not requiring conventional transformations.

# AMPL News

*Free AMPL book chapters*

*AMPL for Courses*

*Extended function library*

*Extended support for “logical” conditions*

- ❖ AMPL driver for CPLEX Opt Studio “Concert” C++ interface
- ❖ Support for ILOG CP constraint programming solver
- ❖ Support for “logical” constraints in CPLEX

*INFORMS Impact Prize to . . .*

- ❖ Originators of AIMMS, AMPL, GAMS, LINDO, MPL
- ❖ Awards presented Sunday 8:30-9:45, Conv Ctr West 101
- ❖ Doors close 8:45!

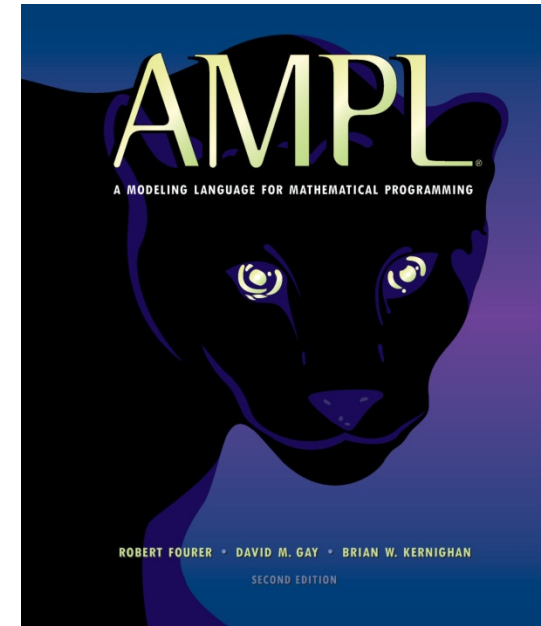
# AMPL Book

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## *Streamlined for quick setup*

- ❖ One-page application form for each course offering
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*... or stop by our booth*

# Extended Function Library

## *AMPL bindings for GNU Scientific Library*

- ❖ Over 300 free open-source functions
  - \* probability distributions: pdf, cdf
  - \* special functions: Bessel, erf, gamma, . . .
  - \* random number generators
- ❖ Easy to “install”
  - \* download `amplgsl.dll` to your AMPL folder/directory

## *Accessible to AMPL*

- ❖ Invoke `load amplgsl.dll`; at start of session
- ❖ Specify function `gsl_ . . .`; for each function needed

## *Accessible to solvers*

- ❖ Apply to variable expressions in objective, constraints
- ❖ 1st & 2nd derivatives provided

# Example

## *hs069 (minimum-cost inspection plan)*

```
function gsl_cdf_ugaussian_P;  
param l {1..4};  
param u {1..4};  
var x {j in 1..4} >= l[j], <= u[j] := 1;  
  
param a := 0.1;  
param b := 1000;  
param d := 1;  
param n := 4;  
  
minimize obj:  
    ( a*n - (b*(exp(x[1])-1) - x[3])*x[4]/(exp(x[1]) - 1 + x[4]) )/x[1] ;  
subject to constr1:  
    x[3] = 2*gsl_cdf_ugaussian_P(-x[2]);  
subject to constr2:  
    x[4] = gsl_cdf_ugaussian_P(-x[2] + d*sqrt(n)) +  
          gsl_cdf_ugaussian_P(-x[2] - d*sqrt(n));
```

*Extended Function Library*

## **Example** *(cont'd)*

### *hs069 solution*

```
model hs069.mod;
data hs069.dat;
load amplgsl.dll;
ampl: option solver knitro;
ampl: solve;
KNITRO 8.0.0: Locally optimal solution.
objective -956.7128867; feasibility error 3.41e-11
10 iterations; 11 function evaluations
ampl: display x;
1  0.0293714
2  1.19025
3  0.233947
4  0.791668
```



*Extended Function Library*

# **Licensing**

*GNU General Public License*

*Suitable for noncommercial uses*

- ❖ Research
- ❖ Stand-alone modeling
- ❖ Open-source development

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- ❖ More permissive open-source licenses
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# Support for “Logical” Conditions

## *Introductory examples*

- ❖ Spatial location
- ❖ Multicommodity transportation

## *Supported “logical” operators*

- ❖ General forms
- ❖ Examples

## *Prospective enhancements . . .*

# Logic Example 1

## *Spatial location*

- ❖ Build  $n$  observation posts
- ❖ Locate at points on an  $n$ -by- $n$  grid
- ❖ Incur equal construction costs

## *Non-interference constraints*

- ❖ No post blocks any other's view
- ❖ along any row, column or diagonal of the grid

*Logic Example 1*

# Mixed-Integer Linear Formulation

*One variable per grid point*

```
param n integer > 0;
var Build {1..n,1..n} binary;
subj to row_conflicts {i in 1..n}:
    sum {j in 1..n} Build[i,j] = 1;
subj to col_conflicts {j in 1..n}:
    sum {i in 1..n} Build[i,j] = 1;
subj to diag1_conflicts {k in 3..2*n-1}:
    sum {i in max(1,k-n)..min(n,k-1)} Build[i,k-i] <= 1;
subj to diag2_conflicts {k in -(n-2)..(n-2)}:
    sum {i in max(k,0)+1..min(k,0)+n} Build[i,i-k] <= 1;
```

*Logic Example 1*

# CP-Style Formulation

*One variable per grid row*

```
param n integer > 0;
var Col {1..n} integer >= 1 <= n;

subj to col_conflicts: alldiff {i in 1..n} Col[i];
subj to diag1_conflicts: alldiff {i in 1..n} (Col[i] + i);
subj to diag2_conflicts: alldiff {i in 1..n} (Col[i] - i);
```

*Logic Example 1*

# Solve with CPLEX or ILOG CP

```
AMPL: model locMIP.mod;  
AMPL: let n := 8;  
  
AMPL: option solver cplex;  
AMPL: solve;
```

```
CPLEX 12.4.0.1: optimal integer solution; objective 0  
26 MIP simplex iterations  
0 branch-and-bound nodes  
Objective = find a feasible point.
```

```
AMPL: model locCP.mod;  
AMPL: let n := 8;  
  
AMPL: option solver ilogcp;  
AMPL: solve;  
ilogcp 12.4.0: feasible solution  
1731 choice points, 1458 fails
```

*Logic Example 1*

# Solve Times

$n$	CP ilogcp	MIP cplex
5	0.02	0.02
10	0.08	0.03
15	0.14	0.08
20	0.14	0.12
25	0.14	0.06
30	0.16	0.03
35	0.28	0.25
40	0.20	0.69
45	0.23	0.12
50	0.34	0.95

*Logic Example 1*

# **Solve Times** (*cont'd*)

$n$	CP ilogcp	MIP cplex
50	0.27	0.94
100	0.39	1.36
150	0.61	5.73
200	1.20	125.86
250	1.11	441.39
300	1.62	1470.29
350	1.56	
400	2.18	
450	2.70	
500	4.15	



# Logic Example 2

## *Multicommodity transportation*

- ❖ Inventory of each product at each origin
- ❖ Demand for each product at each destination
- ❖ Limited shipment capacity of each origin-destination link

## *Minimum-shipment constraints*

- ❖ From each origin to each destination,
- ❖ **either** ship nothing
- ❖ **or** ship at least minload units

*Logic Example 2*

# Mixed-Integer Linear Formulation

*Symbolic data*

```
set ORIG;    # origins
set DEST;    # destinations
set PROD;    # products

param supply {ORIG,PROD} >= 0;    # availabilities at origins
param demand {DEST,PROD} >= 0;    # requirements at destinations
param limit {ORIG,DEST} >= 0;    # capacities of links

param cost {ORIG,DEST,PROD} >= 0;    # shipment cost

param minload >= 0;    # minimum shipment size
```

*Logic Example 2*

# Mixed-Integer Linear Formulation

*Symbolic model (variables and objective)*

```
var Trans {ORIG,DEST,PROD} >= 0;    # actual units to be shipped
var Use {ORIG, DEST} binary;        # 1 if link used, 0 otherwise

minimize Total_Cost:
    sum {i in ORIG, j in DEST, p in PROD} cost[i,j,p] * Trans[i,j,p];
```

*Logic Example 2*

# Mixed-Integer Linear Formulation

*Symbolic model (constraints)*

```
subject to Supply {i in ORIG, p in PROD}:  
    sum {j in DEST} Trans[i,j,p] <= supply[i,p];  
  
subject to Demand {j in DEST, p in PROD}:  
    sum {i in ORIG} Trans[i,j,p] = demand[j,p];  
  
subject to Multi {i in ORIG, j in DEST}:  
    sum {p in PROD} Trans[i,j,p] <= limit[i,j] * Use[i,j];  
  
subject to Min_Ship {i in ORIG, j in DEST}:  
    sum {p in PROD} Trans[i,j,p] >= minload * Use[i,j];
```

*Logic Example 2*

# Mixed-Integer Linear Formulation

*Explicit data independent of symbolic model*

```
set ORIG := GARY CLEV PITT ;
set DEST := FRA DET LAN WIN STL FRE LAF ;
set PROD := bands coils plate ;

param supply (tr):  GARY  CLEV  PITT :=
                    bands  400   700   800
                    coils  800  1600  1800
                    plate  200   300   300 ;

param demand (tr):
                    FRA  DET  LAN  WIN  STL  FRE  LAF :=
bands  300  300  100  75  650  225  250
coils  500  750  400  250  950  850  500
plate  100  100   0   50  200  100  250 ;

param limit default 625 ;
param minload := 375 ;
```

*Logic Example 2*

# Mixed-Integer Linear Formulation

*Explicit data (continued)*

```
param cost :=
  [*,*,bands]: FRA DET LAN WIN STL FRE LAF :=
    GARY 30 10 8 10 11 71 6
    CLEV 22 7 10 7 21 82 13
    PITT 19 11 12 10 25 83 15
  [*,*,coils]: FRA DET LAN WIN STL FRE LAF :=
    GARY 39 14 11 14 16 82 8
    CLEV 27 9 12 9 26 95 17
    PITT 24 14 17 13 28 99 20
  [*,*,plate]: FRA DET LAN WIN STL FRE LAF :=
    GARY 41 15 12 16 17 86 8
    CLEV 29 9 13 9 28 99 18
    PITT 26 14 17 13 31 104 20 ;
```

*Logic Example 2*

# Mixed-Integer Linear Formulation

*Model + data = problem instance to be solved*

```
ampl: model multminship.mod;
ampl: data multminship.dat;
ampl: option solver gurobi;
ampl: solve;
Gurobi 5.0.0: optimal solution; objective 201750
141 simplex iterations
13 branch-and-cut node
ampl: display Use;
Use [*,*]
:      DET FRA FRE LAF LAN STL WIN  :=
CLEV   1   1   1   0   1   1   0
GARY   0   0   1   1   0   1   0
PITT   1   1   1   1   0   1   1
;
```

*Logic Example 2*

# Mixed-Integer Linear Formulation

*Solver choice independent of model and data*

```
ampl: model multminship.mod;
ampl: data multminship.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.4.0.1: optimal integer solution; objective 201750
155 MIP simplex iterations
17 branch-and-bound nodes
ampl: display Use;
Use [*,*]
:      DET FRA FRE LAF LAN STL WIN  :=
CLEV   1   1   1   0   1   1   0
GARY   0   0   1   1   0   1   0
PITT   1   1   1   1   0   1   1
;
```



## Logic Example 2

# Domain-Based Formulation

## Model

```
var Trans {ORIG,DEST,PROD} >= 0;
var SumTrans {i in ORIG, j in DEST}
    in {0} union interval[minload,limit[i,j]];

minimize Total_Cost:
    sum {i in ORIG, j in DEST, p in PROD} cost[i,j,p] * Trans[i,j,p];

subject to Supply {i in ORIG, p in PROD}:
    sum {j in DEST} Trans[i,j,p] <= supply[i,p];
subject to Demand {j in DEST, p in PROD}:
    sum {i in ORIG} Trans[i,j,p] = demand[j,p];

subject to Multi {i in ORIG, j in DEST}:
    SumTrans[i,j] = sum {p in PROD} Trans[i,j,p];
```

*Logic Example 2*

# Domain-Based Formulation

## *Solution*

```
ampl: model multminshipA.mod;
ampl: data multminship.dat;
ampl: option solver gurobi;
ampl: solve;
Gurobi 5.0.0: optimal solution; objective 201750
154 simplex iterations
14 branch-and-cut node
display SumTrans;
SumTrans [*,*]
:      DET   FRA   FRE   LAF   LAN   STL   WIN   :=
CLEV   625   425   425    0   500   625    0
GARY    0     0   375   425    0   600    0
PITT   525   475   375   575    0   575   375
;
```

## Logic Example 2

# Domain-Based Formulation

## Solution

```
ampl: model multminshipA.mod;
ampl: data multminship.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.4.0.1: optimal integer solution; objective 201750
155 MIP simplex iterations
17 branch-and-bound nodes
display SumTrans;
SumTrans [*,*]
:      DET   FRA   FRE   LAF   LAN   STL   WIN   :=
CLEV   625   425   425    0   500   625    0
GARY    0     0   375   400    0   625    0
PITT   525   475   375   600    0   550   375
;
```

## Logic Example 2

# Implication-Based Formulation

## Model

```
var Trans {ORIG,DEST,PROD} >= 0;
var Use {ORIG, DEST} binary;

minimize Total_Cost:
    sum {i in ORIG, j in DEST, p in PROD} cost[i,j,p] * Trans[i,j,p];

subject to Supply {i in ORIG, p in PROD}:
    sum {j in DEST} Trans[i,j,p] <= supply[i,p];
subject to Demand {j in DEST, p in PROD}:
    sum {i in ORIG} Trans[i,j,p] = demand[j,p];

subject to Multi {i in ORIG, j in DEST}:
    Use[i,j] = 0 ==> sum {p in PROD} Trans[i,j,p] = 0 else
        minload <= sum {p in PROD} Trans[i,j,p] <= limit[i,j];
```

*Logic Example 2*

# Implication-Based Formulation

## *Solution*

```
ampl: model multminshipB.mod;
```

```
ampl: data multminship.dat;
```

```
ampl: option solver gurobi;
```

```
ampl: solve;
```

```
Gurobi 5.0.0: Sorry, gurobi cannot handle logical constraints.
```

```
exit code 7
```

```
<BREAK>
```

*Logic Example 2*

# Implication-Based Formulation

## *Solution*

```
ampl: model multminshipB.mod;
ampl: data multminship.dat;
ampl: option solver cplex;
ampl: solve;
CPLEX 12.4.0.1: optimal integer solution; objective 201750
159 MIP simplex iterations
17 branch-and-bound nodes
ampl: display Use;
Use [*,*]
:      DET FRA FRE LAF LAN STL WIN  :=
CLEV   1   1   1   0   1   1   0
GARY   0   0   1   1   0   1   0
PITT   1   1   1   1   0   1   1
;
```

## Logic Example 2

# Disjunctive Formulation

## Model

```
var Trans {ORIG,DEST,PROD} >= 0;

minimize Total_Cost:
    sum {i in ORIG, j in DEST, p in PROD} cost[i,j,p] * Trans[i,j,p];

subject to Supply {i in ORIG, p in PROD}:
    sum {j in DEST} Trans[i,j,p] <= supply[i,p];
subject to Demand {j in DEST, p in PROD}:
    sum {i in ORIG} Trans[i,j,p] = demand[j,p];

subject to Multi {i in ORIG, j in DEST}:
    forall {p in PROD} Trans[i,j,p] = 0 or
    minload <= sum {p in PROD} Trans[i,j,p] <= limit[i,j];
```

*Logic Example 2*

# Disjunctive Formulation

## *Solution*

```
ampl: model multminshipC.mod;
```

```
ampl: data multminship.dat;
```

```
ampl: option solver cplex;
```

```
ampl: solve;
```

```
CPLEX 12.4.0.1:
```

```
logical constraint _slogcon[1] is not an indicator constraint.
```



*Logic Example 2*

# Disjunctive Formulation

*Solution*

```
ampl: model multminshipC.mod;
ampl: data multminship.dat;
ampl: option solver ilogcp;
ampl: solve;
ilogcp 12.4.0: CP Optimizer doesn't support continuous variables.
exit code 1
<BREAK>
```

*Logic Example 2*

# Disjunctive Formulation

*Solution (variables declared integer)*

```
AMPL: model multminshipC.mod;  
AMPL: data multminship.dat;  
AMPL: option solver ilogcp;  
AMPL: solve;  
ilogcp 12.4.0:  
exit code 3  
<BREAK>
```

*... takes longer than you want to wait*

## Logic Example 2

# Disjunctive Formulation

## Solution by CPLEX

```
ampl: model multminshipC.mod;
ampl: data multminship.dat;

ampl: option solver ilogcp;
ampl: option ilogcp_options 'optimizer=cplex';

ampl: solve;

ilogcp 12.4.0: optimizer=cplex
ilogcp 12.4.0: optimal solution
35 nodes, 381 iterations, objective 201750

ampl: display {i in ORIG, j in DEST} sum {p in PROD} Trans[i,j,p];
sum{p in PROD} Trans[i,j,p] [*,*]

:      DET   FRA   FRE   LAF   LAN   STL   WIN   :=
CLEV   625   425   425    0   500   625    0
GARY    0     0   375   425    0   600    0
PITT   525   475   375   575    0   575   375
;
```

*Logic Example 1*

# CP-Style Location Model (*revisited*)

*Solution by CPLEX?*

```
AMPL: model locCP.mod;
AMPL: let n := 8;

AMPL: option solver ilogcp;
AMPL: option ilogcp_options 'optimizer=cplex';

AMPL: solve;

ilogcp 12.4.0: optimizer=cplex

Error: IloAlgorithm cannot extract
       extractables 51, 51, 302, 302, 553 and 553

exit code 1
<BREAK>
```

# Supported CP-Style Operators

## *General forms*

- ❖ Logical
- ❖ Counting
- ❖ Global

## *Examples*

- ❖ Scheduling
- ❖ Assignment
- ❖ Matching
- ❖ Transportation
- ❖ Sequencing

*General Forms*

# Logical Operators

*Unary and binary*

- ❖ *constraint-expr* **and** *constraint-expr*
- ❖ *constraint-expr* **or** *constraint-expr*
- ❖ **not** *constraint-expr*

*Iterated forms*

- ❖ **exists** { *indexing* } *constraint-expr*
- ❖ **forall** { *indexing* } *constraint-expr*

*General Forms*

## **Logical Operators** (*cont'd*)

### *Implication expressions*

- ❖ *if constraint-expr then expr*
- ❖ *if constraint-expr then expr else expr*

### *Implication constraints*

- ❖ *constraint-expr ==> constraint-expr*
- ❖ *constraint-expr ==> constraint-expr else constraint-expr*
- ❖ *constraint-expr <== constraint-expr*
- ❖ *constraint-expr <== constraint-expr else constraint-expr*
- ❖ *constraint-expr <==> constraint-expr*

*General Forms*

# Counting Operators

## *Counting expressions*

- ❖ **count** { *indexing* } ( *constraint* )
- ❖ **numberof** *num-expr* **in** ( *expr1*, *expr2*, ... )

## *Counting constraints*

- ❖ **atmost** *num-expr* { *indexing* } ( *constraint* )
- ❖ **atleast** *num-expr* { *indexing* } ( *constraint* )
- ❖ **exactly** *num-expr* { *indexing* } ( *constraint* )



*General Forms*

## “Global” Operators

### *All-different constraint*

- ❖ `alldiff` { *indexing* } *expr*
- ❖ `alldiff` ( *expr1*, *expr2*, ... )

### *Counting expression*

- ❖ `numberof` *const-expr* `in` ( *expr1*, *expr2*, ... )  
... *consolidate all having same expr-list*

### *Form of *expr1*, *expr2*, ... in list*

- ❖ *expr*
- ❖ { *indexing* } *expr*

*Examples*

# Logical Conditions

*Every job either precedes or follows every other job*

```
subj to NoConflict12
  {i1 in JOBS, i2 in JOBS: ord(i1) < ord(i2)}:
    Start[i2] >= Start[i1] +
      setupTime[i1,i2] - BIG * (1 - Prec[i1,i2]);
```

```
subj to NoConflict21
  {i1 in JOBS, i2 in JOBS: ord(i1) < ord(i2)}:
    Start[i2] >= Start[i1] +
      setupTime[i2,i1] - BIG * Prec[i1,i2];
```

```
subj to NoConflict
  {i1 in JOBS, i2 in JOBS: ord(i1) < ord(i2)}:
    Start[i2] >= Start[i1] + setupTime[i1,i2] or
    Start[i1] >= Start[i2] + setupTime[i2,i1];
```

## Examples

# Logical Conditions (*cont'd*)

*No one should feel isolated within an assigned group*

```
subj to NoIso0 {(i1,i2) in TYPE, j in GROUP}:
  Assign[i1,i2,j] <= upperbnd[i1,i2,j] * Any[i1,i2,j];

subj to NoIso1a {(i1,i2) in TYPE, j in GROUP}:
  Assign[i1,i2,j] >= Any[i1,i2,j];

subj to NoIso1b {(i1,i2) in TYPE, j in GROUP}:
  Assign[i1,i2,j] +
    sum {ii1 in ADJ[i1]: (ii1,i2) in TYPE} Assign[ii1,i2,j]
    >= 2 * Any[i1,i2,j];
```

```
subj to NoIso {(i1,i2) in TYPE, j in GROUP}:
  not ( Assign[i1,i2,j] = 1 and
    sum {ii1 in ADJ[i1]: (ii1,i2) in TYPE} Assign[ii1,i2,j] = 0 );
```

*Examples*

# Cardinality Restrictions

*A warehouse may not serve too many different customers*

```
var Serve {WHSE,CUST} binary;  
  
subj to UDef {i in WHSE, j in CUST, p in PROD}:  
    sum {p in PROD} Trans[i,j,p] <= limit[i,j] * Serve[i,j];  
  
subj to MaxServe {i in WHSE}: sum {j in CUST} Serve[i,j] <= mxsrv;
```

```
subj to MaxServe {i in WHSE}:  
    count {j in CUST} (sum {p in PRD} Trans[i,j,p] > 0) <= mxsrv;
```

```
subj to MaxServe {i in WHSE}:  
    atmost mxsrv {j in CUST} (sum {p in PRD} Trans[i,j,p] > 0);
```

*Examples*

# Matching

*Assign each job to a different machine*

```
var Assign {JOBS,MACHINES} binary;  
subj to OneJobPerMachine {k in MACHINES}:  
    sum {j in JOBS} Assign[j,k] = 1;
```

```
var MachineforJob {JOBS} in MACHINES;  
subj to OneJobPerMachine:  
    alldiff {j in JOBS} MachineForJob[j];
```

*Examples*

# Assignment

*Assign a limited number of jobs to each machine*

```
var Assign {JOBS,MACHINES} binary;  
  
subj to CapacityOfMachine {k in MACHINES}:  
    sum {j in JOBS} Assign[j,k] <= cap[k];
```

```
var MachineforJob {JOBS} in MACHINES;  
  
subj to CapacityOfMachine {k in MACHINES}:  
    numberof k in ({j in JOBS} MachineForJob[j]) <= cap[k];
```

# What's missing (currently)?

## *Domain-based formulations*

- ❖ Sending directly to solver
- ❖ Object-valued variables

## *CP-style formulations*

- ❖ Variables in subscripts

## *Support for additional solvers*

*What's Missing?*

# Domain-Based Formulation

*Variable in arbitrary set of numbers*

```
var SumTrans {i in ORIG, j in DEST}  
    in {0} union interval[minload,limit[i,j]];
```

*Transform in AMPL (current)*

- ❖ Convert to linear MIP
- ❖ Add “SOS” markers used by some MIP solvers

*Send directly to solver (prospective)*

- ❖ Write representation of domain in problem file
- ❖ Let interface decide how to convert for solver
  - \* Semi-continuous variables (CPLEX)
  - \* Arbitrary domains (CP)



*What's Missing?*

## **Variables in Subscripts**

*Return to spatial location problem*

- ❖ Build  $n$  observation posts
- ❖ Locate at points on an  $n$ -by- $n$  grid
- ❖ *Minimize total construction cost*

*Non-interference constraints*

- ❖ No post blocks any other's view
- ❖ along any row, column or diagonal of the grid

*Variables in Subscripts*

# Mixed-Integer Linear Formulation

*Sum of  $n^2$  costs times binary variables*

```
param n integer > 0;
param cost {1..n,1..n};
var Build {1..n,1..n} binary;

minimize TotalCost:
    sum {i in 1..n, j in 1..n} cost[i,j] * Build[i,j];

subj to row_conflicts {i in 1..n}:
    sum {j in 1..n} Build[i,j] = 1;

subj to col_conflicts {j in 1..n}:
    sum {i in 1..n} Build[i,j] = 1;

subj to diag1_conflicts {k in 3..2*n-1}:
    sum {i in max(1,k-n)..min(n,k-1)} Build[i,k-i] <= 1;

subj to diag2_conflicts {k in -(n-2)..(n-2)}:
    sum {i in max(k,0)+1..min(k,0)+n} Build[i,i-k] <= 1;
```

*Variables in Subscripts*

# CP-Style Formulation

*Sum of  $n^2$  conditional terms*

```
param n integer > 0;
param cost {1..n,1..n};
var Col {1..n} integer >= 1 <= n;

minimize TotalCost:
    sum {i in 1..n, j in 1..n} if Col[i] = j then cost[i,j];

subj to col_conflicts: alldiff {i in 1..n} Col[i];
subj to diag1_conflicts: alldiff {i in 1..n} (Col[i] + i);
subj to diag2_conflicts: alldiff {i in 1..n} (Col[i] - i);
```

*Variables in Subscripts*

# CP-Style Formulation

*Sum of n costs*

```
param n integer > 0;
param cost {1..n,1..n};
var Col {1..n} integer >= 1 <= n;

minimize TotalCost:
    sum {i in 1..n} cost[i,Col[i]];

subj to col_conflicts: alldiff {i in 1..n} Col[i];
subj to diag1_conflicts: alldiff {i in 1..n} (Col[i] + i);
subj to diag2_conflicts: alldiff {i in 1..n} (Col[i] - i);
```

... *“variable in subscript”*

*Variables in Subscripts*

# Domain-Based Formulation

*Variable in arbitrary set*

```
var Serve {CLI} in LOC;  
var Open {LOC} binary;  
  
minimize TotalCost:  
    sum {i in 1..mCLI} srvCost[i,Serve[i]] +  
    bdgCost * sum {j in 1..nLOC} Open[j];  
  
subject to OpenDefn {i in 1..mCLI}: Open[Serve[i]] = 1;
```

*What's Missing?*

## **Additional Solvers**

*ILOG CP is a good start*

- ❖ Allows experimentation with a mature general-purpose CP solver
- ❖ Permits some comparisons to a MIP solver
- ❖ . . . *but IBM has a modeling language tailored to CP*

*AMPL's strength is solver-independence*

- ❖ Other solvers can benefit from this technology
- ❖ Various possibilities under consideration
  - \* Other constraint programming solvers
  - \* LINDO Global
  - \* SCIP
  - \* LocalSolver
  - \* . . .

# How to Get the CP Interface

*If you have an IBM CPLEX license from us*

- ❖ Log in to your account at the AMPL download site
- ❖ Click on an ilogcp link to get a bundle of the needed files

*If you have a AMPL academic license from us*

- ❖ Join the IBM Academic Initiative
- ❖ Send info@ampl.com a request to add free 1-year licenses for the CPLEX and ILOG CP solvers

*Or request a free 30-day AMPL trial*

- ❖ Write in ILOG CP as one of the solvers to include





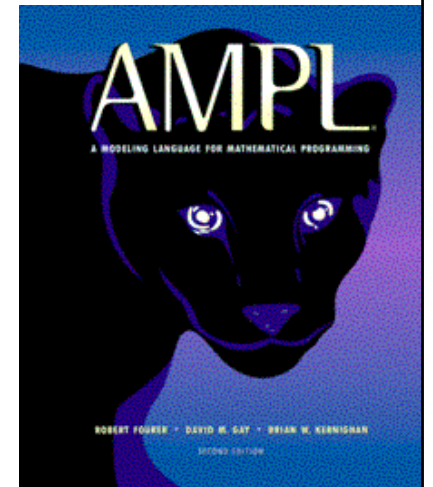
# SolverStudio

## AMPL Modeling in Excel

**SolverStudio** is an integrated Excel add-in that makes it easy to develop and deliver AMPL optimization models using the familiar Excel environment. SolverStudio adds a text editor to Excel that allows an optimization model to be created using AMPL and then embedded and saved within a spreadsheet. SolverStudio also provides an integrated data editor that allows model data (AMPL parameters and sets) to be stored and edited on the spreadsheet. SolverStudio's *Solve* button runs the AMPL model while seamlessly managing data transfers with the spreadsheet. AMPL models can be solved locally, or in the cloud using NEOS. As well as working with AMPL, SolverStudio also supports the GAMS, PuLP and Gurobi Python modelling languages, amongst others.

SolverStudio is being developed and supported by Andrew Mason at the Department of Engineering Science, University of Auckland, New Zealand.

<http://solverstudio.org>



**Solver  
Studio  
for Excel**

# SolverStudio

SolverStudio for AMPL.xlsx - Microsoft Excel

Home Insert Page Layout Formulas Data Review View Developer Team

Get External Data Refresh All Sort Filter Reapply Advanced Text to Columns Remove Duplicates Outline Data Analysis Solver

Show Model Show/Hide Data Edit Data Show Data in Color Solve Model About SolverStudio

Ingredients	Costs	Contributes					Amount
		Protein	Fat	Fibre	Salt	OneCan	
Chicken	0.013	0.1	0.08	0.001	0.002	1	0
Beef	0.008	0.2	0.1	0.005	0.005	1	10
Mutton	0.01	0.15	0.11	0.003	0.007	1	0
Rice	0.002	0	0.01	0.1	0.002	1	0
Wheat bran	0.005	0.04	0.01	0.15	0.008	1	0
Gel	0.001	0	0	0	0	1	90

```
set INGREDIENTS;
set REQUIREMENTS;
param Cost {INGREDIENTS};
param Contributes {REQUIREMENTS, INGREDIENTS};
param Lower {REQUIREMENTS};
param Upper {r in REQUIREMENTS} >= Lower[r];
default Infinity;
var Amount {INGREDIENTS} >= 0;

minimize TotalCost:
  sum {i in INGREDIENTS} Cost[i] * Amount[i];
subject to MeetRequirements {r in REQUIREMENTS}:
  Lower[r]
  <= sum {i in INGREDIENTS} Contributes[r, i] * Amount[i];
  <= Upper[r];

data SheetData.dat; # Get data from the sheet

option solver cplexamp;
solve;

display Amount > Sheet; # Write the solution to the sheet
```

Model Output

AMPL Version 20080102 (x86\_win32)  
CPLEX 11.0.0: optimal solution; objective 0.17  
2 dual simplex iterations (0 in phase I)  
## AMPL run completed.

Row 19, Column 35

Average: 16.6666667 Count: 6 Min: 0 Max: 90 Sum: 100

- Build & solve AMPL models using Excel
- Solve AMPL models in the cloud using NEOS.
- Free download: <http://solverstudio.org>

# SolverStudio

SolverStudio for AMPL.xlsx - Microsoft Excel

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Get External Data Refresh All Sort Filter Reapply Clear Reapply Advanced Text to Columns Remove Duplicates Outline Solver Data Analysis Solve Model Show Model Edit Data Show/Hide Data Show Data in Color About SolverStudio SolverStudio

INGREDIENTS Chicken

Ingredients	Costs	Prote
Chicken	0.013	0.1
Beef	0.008	0.2
Mutton	0.01	0.1
Rice	0.002	0
Wheat bran	0.005	0.0
Gel	0.001	0

SolverStudio - Data Items Editor

Name:	Cell Range:	Index Range(s):	Value if Missing:
<Add New Data Item >			
Amount	J4:J9	INGREDIENTS	<Report an error >
Contributes	D4:H9	REQUIREMENTS, INGREDIENTS	<Report an error >
Cost	C4:C9	INGREDIENTS	<Report an error >
INGREDIENTS	B4:B9		
Lower	D13:H13	REQUIREMENTS	<Report an error >
REQUIREMENTS	D3:H3		
Upper	D12:H12	REQUIREMENTS	<Report an error >

Name: Cell Range: Index Range(s): Unknown Indices return

INGREDIENTS B4:B9 an error

Delete Data Item Update Data Item Cancel

This dialog allows you to define cell ranges ('data items') on your spreadsheet that you want to use in your optimisation model. You can define sets of items, indexed lists (a cell range containing one entry for each value in an associated index range) and indexed tables (which need 2 index ranges).

Count: 6 100%

- Easily define params & sets on spreadsheet
- Automatic data exchange with model
- Free download from <http://solverstudio.org>